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UNDERSTANDING BAYES' THEOREM AND ITS APPLICATION IN JUDICIAL DECISION MAKING

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ABSTRACT
The Bayes' theorem is a mathematical formula to gauge and describe the probability of an event by employing prior knowledge and evidence relevant to the event. The objective of the present study was to understand the Bayesian theorem and its application in judicial trials by deploying doctrinal research methodology. After consulting authoritative writings of prominent researchers and judicial decisions, study found that the Bayesian Probability in legal context used in odds version, likelihood ratio and in the form of Bayesian networks. The study also found that the application of the theorem in judicial proceedings was controversial since various researchers condemned, and numerous analysts advocated its application in real time court cases. Moreover, the study found that the theorem has been and advocated to be used to measure the probative force of statistical and non-statistical evidence, and to infer the causes of any event by observing its effects. It is expected that the present study will enable the legal fraternity to understand the working mechanism and various uses of Bayes' theorem in legal context.
Keywords: Application of Bayesian theorem, Bayesian Probability, Judicial decisions, Uses of Bayesian theory in judicial trials.

1. INTRODUCTION
The primary responsibility of judicature in criminal cases is to determine the guilt or innocence of accused. The judges resolve the guilt or innocence hypothesis about accused by evaluating the evidence which may be ambiguous, incomplete, and uncertain. Studies have demonstrated that decision makers in every discipline may suffer from number of biases when they make decisions based on evidence, hence it is necessary that their decisions must be impartial and robust (Jaunzemis et all, 2019, p. 26). The same is true regarding criminal trials since their success largely depends upon the ability of the decision makers to reasoning with evidence accurately, rationally and impartially. Like other disciplines, the judicial decision makers may also commit reasoning errors while evaluating evidence and these errors may have severe consequences. Three kinds of approaches or models i.e. argumentative approach, the story model and probabilistic approaches have been developed by various researchers as a tool to prevent reasoning errors while evaluating judicial evidence Argumentative approach examines the arguments and counter arguments about a particular ultimate probanda, the story model is concerned with constructing and comparing various stories about what happened and the Probabilistic methods involves the application of the theory of probability and it revolves around demonstrating a link between evidence and hypothesis (Verheij, 2014, p. 307).

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As far as probability is concerned, it is a tool to estimate the uncertainty about the past or future occurrences. Allen (2017) discussed four types of probability: relative frequency, classical or logical probability, propensity related probability and Bayesian probability. He elaborated that probability based on relative frequency describes the statement of the likelihood of a particular outcomes in a long run of trial and it indicates the size of subset in a set of interest. Likewise the classical or logical probability involves the logical explication of probability. On the other hand, propensity theory based on probability is refereed to understanding the things like radioactivity which is not the subject matter of trial normally. Similarly, Bayesian probability is concerned with ascertaining the degree of belief about the occurrence of past or future event and subjective and objective probability are its two types (Allen, 2017, p.134).

The application of mathematics in law is associated with Professor Coase who carried out the economic analysis of law depending on the theory that all human being are rational calculator of their decisions (Guerra-Pujol, 2011, p. 210). Similarly, the Bayesian probability has also been introduced in law to avoid logical contradictions and to advance rational fact finding (Hunt & Mostyn, 2020, p. 75). It is significant to mention that Bayesian probability requires the decision makers to update their belief about a particular hypothesis after observing the evidence and this feature of Bayesian probability makes it suitable for judicial proof (Berger, 2015, p.9). However, the Bayesian theorem has been proved tricky for the legal fraternity as its application involves mathematical equations which is alien for the legal fraternity. Various researchers have discussed, analyzed, approved and disapproved the application of Bayesian probability in law, however; these studies did not elaborate the working mechanism, merits and demerits of its application and for what purposes the Bayesian probability has been recommended to be used in judicial trials. The study will significantly attempt to bridge this gap since its primary purpose is to understand the application of the Bayesian probability in judicial proceedings. The present study revolves around to probe three research questions; Firstly, What is Bayesian probability and how is it applied? Secondly, What are the merits and demerits of its application in judicial trials? and lastly for what purposes, has the Bayesian probability been used in judicial trials? The present study, other than introductory part, has four sections. The concept, mechanism and different forms of the Bayes theorem have been elaborated in second part of the study. Third part contained positive and negative dimensions of the theorem and its legal application, the fourth section discusses the various uses of the theorem in law and the last section concludes the study.

2. Bayesian Probability: Mechanism and Forms

Bayes' theorem is a technique to compute probability of the happening or non-happening of past or future events. The basic terms which the theorem involves include hypothesis, evidence, odds, likelihood ratio, prior probability, posterior probability and conditional or posterior probability. The term hypothesis in Bayesian theorem stands for a Boolean, statement whose truth value is unknown and is to be determined In addition, the unknown truth value of the hypothesis can never be determined with certainty In the judicial context, the hypothesis may be divided into an ultimate hypothesis (the hypothesis that accused is the person who committed the offense, a source level hypothesis (DNA) found at the crime scene was that of the accused) and the alternative hypothesis which is the opposite of the ultimate hypothesis Similarly, the term evidence refers to any true statement which offers support to any hypothesis (Fenton et al., 2016). Likewise, the term "odds" refers to the ratio of happening of an event divided by non-happening of the event. Odds are of two types; prior and posterior. Prior odds gauge the comparative degree of belief between prosecution and defense hypothesis before the evidence has been incorporated and the posterior odds are meant to measure the strength of belief after the evidence has been incorporated. Similarly, the term "likelihood ratio" indicates the summary of the effect of the evidence on hypothesis. It is an important indicator of the probative value of evidence which shows the probabilities granted to the new evidence by each of the competing hypothesis. Likewise, the posterior probability (also called updated probability and conditional probability) refers to the probability of hypothesis after observing evidence. On the other hand, the prior probability stands for the probability of hypothesis before observing any evidence.

The Bayesian theorem is a technique to compute the probability of an event and it is generally applied in algebraic form, odds forms and in Bayesian networks. The algebraic form of the theorem takes the following form:

$$\boxed{Pr(A|B) = ([Pr(B|A)] \times [Pr(A)]) \div Pr(B)}$$

The algebraic expression of the theorem may be described in words by dividing it into five parts. Firstly, the term at the extreme left side, Pr(A|B), indicates the conditional probability (or posterior probability) of event A, given the occurrence of event B. Secondly, the right-hand side of the equation is a fraction: the numerator contains two parts, Pr(B|A) × Pr(A), while the denominator consists of one term, Pr(B). Thirdly, the term Pr(B|A), talks about the conditional probability of event B, given the occurrence of event A. Fourthly, Pr(A), refers to the prior probability (or unconditional probability) of event A, that is, the probability of A in the absence of any information about event B. Lastly, Pr(B), is the prior probability (or unconditional probability) of event B in the absence of any information about event A (Pujol, 2011, p. 7).

The second form of the application of the Bayesian theorem is the odds form.The application of the Bayesian theorem in its odds form in legal context in the following words. To his mind, the judge's ultimate task of evaluation of court evidence is to obtain the posterior probability of an accused's guilt on the basis of evidence and he describes the theorem in odds form in the following words;

$$\boxed{P\ (\text{guilt/evidence})\ldots\ldots\ldots\ldots\ldots 1}$$

He says that P (A/B) indicates the conditional probability of an uncertain event A on the basis of the information B. He adds that it is the direct way and not appropriate to assess the guilt hypothesis of the accused. He recommends using odds form of Bayesian theorem to do that through the following equation:

$$\boxed{P\ (G/E)\ldots\ldots\ldots 2}$$

The above mentioned two equations may be described in words as follow:

$$\boxed{\text{Posterior odds} = \text{prior odds} \times \text{likelihood ratio.}}$$

The first term, in above equation on the right side indicates the prior odds which measure the comparative degree of belief of prosecution and defense hypothesis before the evidence has been observed. The judge may have reasonable values in his mind on the basis of the previous evidence. The second term, the likelihood ratio indicates the probabilities assigned to the new evidence in the light of the two competing hypothesis. By multiplying the likelihood ratio with prior odds, it will result in the posterior odds of hypothesis (David, 2002, p. 3).

Likewise, the third form of the application of the Bayesian probability in law is the Bayesian networks. A Bayesian network (also called a causal model) is a directed graphical model for representing conditional independencies between a set of random variables. It is a mixture of probability theory and graph theory, and affords an effective apparatus to handle two complications that occur through applied mathematics and engineering—uncertainty and complexity (Sun & Zhang, 2006, p. 125). The Bayesian network is constructed with arc and nodes where an arc from node A to B exhibits that A causes B (Jensen, & Nielsen, 2007). These networks have been proposed for legal decision making (Keppens, 2013). The various researchers have proposed idioms-based networks for reasoning with different types of evidence related to different events and aspects of a case (Neil et all, 2000, Fenton et all, 2013). These networks may enable judges to envisage and model dependencies amongst various hypotheses and evidence and to estimate the revised probability beliefs about all uncertain factors when any piece of new evidence is presented (Fenton et all, 2013).

3. Deleterious Aspects of Bayesian Probability

The major criticism on the use of Bayesian theorem in judicial decision making lies in the point that the Bayesian applications will complicate the legal reasoning. Proof model based on mathematics may be very much clear to a seasoned mathematicians or logicians but it will prove to be useless for legal fraternity until it is not properly and efficiently explained to it. Bayes theorem may carry formal complexity in the legal reasoning which will result in an inefficient, confusing and counterproductive implementation (Walker, 2006, p. 1693). The parallel aspect has been highlighted by Field (2013) who maintained that it was a difficult task for average decision-makers to calculate and compute the probabilities of different events especially the conditional probability by using Bayes theorem. Study by Cohen (1981) supplemented that the use of Bayesian theorem for in judicial process should be avoided since the Bayesian theorem is not in line with how human mind makes decisions. In addition, author believed that Bayesian inference became one sided because it does not address the different level of credibility. Allen (2017) also criticized that the probability theory in general and Bayesian probability in particular cannot elucidate the inferential process and the structure of trials in common law countries while exercising Bayesian Theorem during judicial trials. Another negative aspect of the judicial application of Bayesian theorem is the problem of appropriate reference class. Generally, its application requires calculating the prior probability which is subsequently used to measure the likelihood ratio. In fact the "reference class" in probability theory refers to measure the prior probability and in legal context, it stands for the class which is selected as a basis to evaluate the evidence (Allen, 2017).

Reference class has embattled in research fields by different scholars. Allen & Pardo (2007) contended that the reference class problem where objective probability based on a specific piece of evidence is a part, cannot establish the probative value of that piece of evidence. Secondly, it is an epistemological limitation to establish the probative value of a specific piece of evidence as having different possible and opposite pointed directions. Thirdly, there is no reliable and empirical data for reference class. There is no trustworthy empirical evidence establishing that Bayesian probability is a suitable tool for better decision making. The subjectively selected "reference class" is not supportive in assessing the probative value of evidence since the probative force of any evidence can neither be equated with reference class nor with the difference between prior and posterior probabilities Similarly, this probability is also measured as an ineffective and unable to measure the combined evidentiary value of various pieces of evidence related to different issues. For instance, the combined probative force of various pieces of evidence related to different events cannot be measured with subjective or objective probability since both suffer from "reference class" problem (Allen, 2007).

One drawback of the application of Bayesian theorem is bar on double usage of evidence in judicial trial, as emphasized in study by Tribe. This theorem requires the decision makers to assign initial probabilities as hypothesis which they do so by using case specific evidence, their personal knowledge and judicial experience. When evidence has been used for assigning initial probabilities, the theorem does not allow its usage for the second time and consequently the same evidence cannot be used for any other purpose. He argued that the restriction on the double usage of evidence is not aligned with the trial norms (Tribe, 1970).

On the same line of reasoning, the inappropriateness of the theorem is underlined as another drawback. In Regina versus Dennis John Adam (1996), the appellant court pointed out that the use of Bayesian theorem for assessing the evidentiary value of evidence would bring unnecessary complexity in judicial trials. Secondly, the evaluation of judicial evidence is the function of the jury which definitely judges perform depending upon their personal experience, their knowledge about world around and their common sense. The trespass on jury functions may possibly be committed if any mathematical formula is applied to evaluate judicial evidence. Similarly, in Nulty & Ors versus Milton Keynes Borough Council (2012) case, the appellant court also precluded the application of Bayes theorem in judicial trials by considering it 'over formulaic' and 'intrinsically unsound'.

The likelihood ratio (LR) with Bayesian theorem in judicial decision making is taken as another issue. The most common and simplest use of LR (the single evidence for a single) is very complex. The formation of LR is extremely difficult since it is made up of multiple hypotheses and its formation requires the decomposition of hypothesis and each piece of evidence and possible only if all the pieces of evidence are joined together. The provision of reliable measure of the probative value of evidence regarding alternative hypotheses is the role of likelihood ratio in the theorem. Actually, the probative value of alternative hypothesis 1 and hypothesis 2 means that after observing the evidence, the posterior probability of hypothesis 1 is greater than the prior probability of the hypothesis 1 and similarly if after observing the evidence about hypothesis 1, there is no difference between prior and posterior probability of the same hypothesis, it means that the evidence has no probative value. However, probative value in terms of Bayesian theorem means the hypothesis 1 and the hypothesis 2 are mutually exclusive and exhaustive. If hypothesis 1 and 2 are mutually exclusive but not exhaustive, then the likelihood ratio will be silent on the relationship between prior and posterior probability and if two hypotheses are not mutually exclusive and both may be true, the likelihood ratio becomes worthless (Fenton et al., 2016).

The concept of LR is used by the forensic expert because LR can be calculated without any prior probability of the hypothesis (Press, 2009). On the other hand, Fenton et al (2016) are of the view that it is wrong to assume that LR can be calculated without prior probability of a hypothesis. They argued that this approach is the result of confusion and misunderstanding of the theorem and they offered two reasons for such misunderstand. Firstly, LR itself speaks nothing about the hypothesis being true: posterior probability can only be calculated if the prior probability is known. Hence, it is not possible to calculate LR without knowing the prior probability. Secondly, the specification of the LR im prosecution and defense hypotheses is impossible until something is known about the two hypotheses. They believed that LR and priors are all conditioned on some background knowledge which is necessary for correct calculation of posterior probability in addition to this, concluded that different and contradicting likelihood ratio for the same hypothesis would be formed without using background knowledge or prior, hence, prior probability would be calculated to measure the LR of any hypothesis.

On the other side, study by Tribe did not approve the application of Bayesian probability in judicial trials on the ground that its application is a threat to the fundamental doctrine of criminal law i-e; presumption of innocence. The Principle "Accused is Presumed innocent till guilty proven" is drawn at the beginning of the trial and is rebutted at the end of the trial when whole evidence has been observed by fact-finders. On the other hand, the presumption of innocence may be rebutted at the beginning of a trial, or during the trial or at the conclusion of trial when Bayesian theorem is used to draw conclusion about guilt or innocence hypotheses of an accused (Tribe, 1970).

4. Beneficial Dimension of Bayesian Probability

The Bayesian probability methods offer a chart expressing the uncertainty in decision making process. This chart is capable of exhibiting the probative force of not only single piece of evidence but also the probative value of whole evidence which is an effective way to make decisions on the basis of evidence. Similarly, the Bayesian theorem is frequently used in various disciplines including law to evaluate statistical evidence. However, the theorem may even be used to evaluate non-statistical evidence in legal cases.

It is assumed that the mathematical model of decision making based on Bayes theorem may be exercised in a cases to evaluate and reflect the degree of belief of the non- statistical evidence which rises suspicion about an accused (Berger, 2015).

Transparent, logical and fair evaluation of evidence presented during judicial proceedings is the pre-requisites of the doctrine of fair trial. In addition, the fair trial doctrine also requires that the decisions by courts should be given on the basis of evidence that the fact finders assessed the probative force of evidence. The transparent, fair and logical evaluation of evidence is possible if all the plausible issues and disputed questions of law and fact are appropriately underlined during the trial.

It is acknowledged that the Bayes theorem is a good strategy to meet essentials of the fair trial doctrine, as it could highlight and make clear various contested issues in judicial proceedings which will otherwise may remain ambiguous and uncertain in the traditional set up of judicial trials. Bayesian probability is the logic of rational inference that ensures transparency to the formulation and analysis of difficult problems. Same study projected Bayes' theorem as a good tool to evaluate the probative force of single piece of evidence on hypothesis but also it is handy device to measure the probative force of various pieces of evidence as combined evidence (Dawid, 2002).

In current era, DNA evidence is frequently taken into consideration in criminal trials to identify an accused. However, the interpretation of the DNA evidence is a complicated and difficult task requiring transparent and reliable skills. It is reasoned that the complex understanding of DNA evidence creates the possibility of errors in the interpretation which may lead to miscarriage of justice. Probabilistic methods may be exercised in general and Bayesian probability in particular to interpret the DNA evidence due to their transparent mechanism since the criminal justice system demands transparency (Berger, 2015).

The Bayesian probability is considered as a handy tool to get rid of prejudices while constructing decisions based on evidences. The literature on decision making by human being produced in psychology rank the cognitive unfairness as the fundamental reason of erroneous decision making by the human beings. The same is given weightage where judges may precede erroneously and where avoidances of such cognitive prejudice is mandatory. Various researchers have proposed to use the Bayes theorem in judicial decisions on the ground that it offers an objective method to evaluate the evidence and its impact on decision made by the judges (Fields, 2013).

5. Legal Perspective of Bayesian Theorem

Bayesian Theorem and its application in judicial trials is a question of great debate amongst many academic scientists as various do not approve its use in judicial proceedings and many analysts supported its legal perspective. This section is devoted to address the third research question i-e; the various uses of Bayes theorem in judicial proceedings as discussed by prominent researchers.

5.1. Identification Evidence

The major responsibility ask of judges in criminal proceedings is to determine criminal liability of offenders for the act committed. This task is to be achieved by duly observing the different procedures of as fall under the domain of Law of Evidence and criminal law. Number of ways like eyewitnesses' testimony, finger prints, foot prints, clothes, blood, weapons, soil and DNA evidence may be taken into consideration. Fenton et al., (2014) considered that trait evidence including DNA, is the most important and reliable evidence to identify the accused. They further maintained that the judicial decision makers may deploy the Bayesian theorem (in likelihood ratio) to assess and evaluate the effects of trait evidence related to the identity of accused including his DNA. However, the courts have shown their reluctance to use Bayes theorem to evaluate certain type of trait evidence. For instance, in R v T (2009), the court did not approve the use of Bayes' theorem and likelihood ratio to express the probative value of foot wear evidence. The court ruled that the use of formulas to calculate probabilities and reason about the value of evidence was inappropriate in areas such as footwear mark evidence. In addition, some analysts also admitted that the correct presentation of match evidence in the court is very perplexing and dangerous since the match testing is prone to errors.

However, Fenton, et al., (2014) argued that the Bayes' theorem may be used to evaluate trait evidence despite the decision in R v T. Nevertheless, they associate two major challenges with the application of Bayes' theorem in judicial proceedings; the first challenge relates to ensuring the correctness of probability calculations and the second challenge is about the explanation of the meaning and application of Bayesian probability to an ordinary person (Fenton et al.,2014). Another Study proposed that the second challenge might be addressed by using event tree annotated with frequency values for probabilities of events (Gigerenzer, 2003). Likewise, Fenton et al., (2014) are of the view that the first challenge is easy to handle and even it may be handled manually. They suggested using a generic frame work to tackle the challenges associated with presentation of match evidence in terms of Bayes' theorem. They advocated the use of graphical Bayesian network to obtain the probabilities since the application of Bayesian networks is automatic and easy to apply. They claimed that the networks developed by him are a good technique for the production and examination of match evidence.

5.2. Bayesian to infer cause from effects

Many renowned analysts considered Bayes Theorem as an effective tool to infer the cause from effects. Evaluating the inferred causes after observing the effects may become the major task in criminal trials. For instance, the post mortem report in a murder case is meant to express the cause of death, the time of death and the nature of weapon used to cause death. Dawid et al., (2016) pointed out that legal fraternity is more inclined towards understanding the causes of effects as compare to quantitative and qualitative scientists whose interest are to study the effects of causes. Probabilistic reasoning is relatively rich and mature to comprehend the effects of causes through experiments and observations as compare to the methods of understanding the causes of effects. In legal context, they suggested using a model based on Bayesian probability (their approach is called prior-to-posterior inference) to infer the cause of effects. They applied the model to understand the relationship between personalist judgments and the empirical evidence which informs them, making use of exchangeability considerations to relate personal probabilities to frequencies observed in data.

5.3. Bayesian To Assess the Combined Effect of Statistical Evidence

Many researchers have advocated the use of Bayesian theorem to organize, analyze and evaluate the combined effects of different pieces of evidence derived from different sources. It is a difficult task in criminal cases to properly evaluate and organize statistical evidence in court cases and there are many instances of miscarriage of justice due to wrong interpretation of statistical evidence in the past. For instance, if different data had been combined correctly and in statistically justifiable way, the result in Lucia' case would have been different (Meester et all, 2006). Due to the possibility of misinterpretation of statistical evidence, it is proposed that to assess the combine effects of evidence use of Bayesian theorem is effective. It can be used for instance, to organize and examine the intricate connection between diverse pieces of evidence derived from different sources like fiber analysis and bloodstain analysis, using computational systems based on Wigmore's work to model and organizing different pieces of evidence (Dawid, 2002). On the same line of reasoning, Fenton et al., (2016) proposed using the Bayesian probability to organize and analyze the combined effects of statistical evidence coming from diverse sources in murder cases. He explained that statistical evidence (CCTV, DNA finger prints, foot prints) in murder cases may originate from multiple sources and it is the most crucial step to combine the different pieces of statistical evidence to measure their total impact on the hypothesis and the Bayes theorem is ideal for integrating all type of statistical evidence in court case. Similarly, Fienberg and Kaye recommended using Bayesian probability to integrate various pieces of similar fact evidence to gauge their combined effects on hypothesis (Fienberg, & Kaye, 1991).

5.4. Bayesian To Retracted Statements

Field (2013) purposed that the Bayesian theorem might be used to evaluate recanted statements of witnesses (a court statement in which the witness publicly retracts from his previous statement). Author pointed out that procedural law or policy sometimes makes it difficult for the order of new trial due to retracted statement. In such circumstances, she proposes to use Bayesian theorem to assess the authenticity or credibility of such testimony. She proposes to use different priors to evaluate such testimony which will result in a variety of posterior probabilities. Subsequently, the judge may choose one posterior probability having the highest score out of other posterior probabilities She claims that Bayesian theorem will not only bring consistency and accuracy in such cases but also the theorem will be helpful in avoiding cognitive bias and injustice (Field, 2013).

5.5. Fingerprint, Handwriting, speech recognition, ear prints, footprints etc.

Similarly, various researchers have advocated the use of Bayesian theorem to evaluate the evidence which has been derived with the help of modern devices. For instance, Aitken and Taroni (2004) suggested using Bayesian probability to analyze the fingerprint evidence by analyzing the source fingerprint and target fingerprint. They maintained that criminals may leave their finger impression on the crime scene which the law enforcement agencies picked up with modern devices. Afterwards, these finger impressions are compared with the specimen fingerprint taken from the suspect and the experts make comparison to see whether both match each other or not. They pointed out that Bayesian probability may be used to carry out the comparison of two finger impressions. On the other hand, Alberink et al., (2014) recommended to use automated version of Bayesian theorem (based on likelihood ratio) for the identification and the comparison of the fingerprints, fibers, paint, speech handwriting, footprints and ear prints etc.

CONCLUSIONS

Probability is a tool to measure the uncertainty of belief about the past or future facts. The relative frequency, classical or logical probability, propensity probability and Bayesians probabilities are the types of probability. The Bayesian theorem estimates the uncertainty of belief about past or future events by setting out hypothesis, organizing evidence, assessing the value of each piece of evidence and measuring the weight of belief with and without evidence. This basic mechanism of the theorem is applied in three forms; in algebraic form, in odds form and in Bayesian networks. The application of the theorem in court cases is controversial. Various researchers have criticized its application on the ground that it is complex to understand, it will bring complexity in legal reasoning, difficulty to find appropriate reference class and likelihood ratio, double usage of evidence, trespass on jury functions and its threat to presumption of innocence. On the other hand, the application of the theorem is advocated on the ground that it can tackle judges' cognitive bias; it provides graphical mechanism to understand the evidence and to assess its probative value. In addition, the Bayesian theorem is inevitable in judicial trials since it is a handy tool to evaluate scientific evidence and can highlight all the potential issues in judicial proceedings. Despite the severe criticism on the application of the Bayes' theorem in law, it has been used for variety of purposes. The theorem has been used to evaluate statistical and non-statistical evidence (finger print, foot print, thumb impression, DNA evidence, trait evidence and witnesses' testimony). Similarly, the theorem is considered as a good device to infer the cause of an event after observing the effect of the event and finally to assess the probative value of evidence.

REFERENCES

Alberink, I., de Jongh, A., & Rodriguez, C. (2014). Fingermark evidence evaluation based on automated fingerprint identification system matching scores: the effect of different types of conditioning on likelihood ratios. Journal of forensic sciences, 59(1), 70-81. doi: 10.1111/1556-4029.12105. Epub 2013 Nov 1. PMID: 24180303.

Allen, R. J. (2017). The nature of juridical proof: Probability as a tool in plausible reasoning. *The International Journal of Evidence & Proof*, *21*(1-2), 133-142. https://doi.org/10.1177/1365712716674794

Allen, R. J., & Pardo, M. S. (2007). The problematic value of mathematical models of evidence. *The Journal of Legal Studies*, *36*(1), 107-140. https://doi.org/10.1086/508269

Aitken, C., & Taroni, F. (2004). *Statistics and the evaluation of evidence for forensic scientists*. John Wiley & Sons. https://www.wiley.com/en-us/Statistics+and+the+Evaluation+of+Evidence+for+Forensic+Scientists%2C+2nd+Edition-p-9780470011225

Alberink, I., de Jongh, A., & Rodriguez, C. (2014). Fingermark evidence evaluation based on automated fingerprint identification system matching scores: the effect of different types of conditioning on likelihood ratios. *Journal of forensic sciences*, *59*(1), 70-81. doi: 10.1111/1556-4029.12105. Epub 2013 Nov 1. PMID: 24180303

Berger, D. R. H. J. (2015). *Improving legal reasoning using Bayesian probability methods* (Doctoral dissertation, Queen Mary University of London). https://qmro.qmul.ac.uk/xmlui/bitstream/handle/123456789/8914/Berger_Daniel_PhD_190515.pdf?sequence=1

Cohen, L. J. (1981). Can human irrationality be experimentally demonstrated? Behavioral and Brain Sciences, 4(3):317-331.
https://www.cambridge.org/core/journals/behavioral-and-brain-sciences/article/abs/can-human-irrationality-be-experimentally emonstrated/E7A6D9F9A380C2E15DF597CF6D17CA46

Dawid, A. P., Musio, M., & Fienberg, S. E. (2016). From statistical evidence to evidence of causality. International Society for Bayesian Analysis, 11(3): 725–752. DOI: 10.1214/15-BA968

Dawid, A. P. (2002). Bayes's theorem and weighing evidence by juries. In Bayes's Theorem.113: 71-90.

Fienberg, S. E., & Kaye, D. H. (1991). Legal and statistical aspects of some mysterious clusters. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, *154*(1):61-74 https://www.jstor.org/stable/2982695

Fenton, N., Neil, M., & Berger, D. (2016). Bayes and the law. *Annual review of statistics and its application*, *3*:51-77. https://www.annualreviews.org/doi/10.1146/annurev-statistics-041715-033428

Fenton, N., Neil, M., & Lagnado, D. A. (2013). A general structure for legal arguments about evidence using Bayesian networks. *Cognitive science*, *37*(1), 61-102. https://doi.org/10.1111/cogs.12004

Fienberg, S. E., & Kaye, D. H. (1991). Legal and statistical aspects of some mysterious clusters. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, *154*(1), 61-74. https://www.jstor.org/stable/2982695

Fenton, N., Neil, M., & Hsu, A. (2014). Calculating and understanding the value of any type of match evidence when there are potential testing errors. *Artificial Intelligence and Law*, *22*, 1-28. http://dx.doi.org/10.1007/s10506-013-9147-x

Fields, K. L. (2013). Toward a Bayesian analysis of recanted eyewitness identification testimony. *NYULRev.*, *88*:1769-1801. https://www.nyulawreview.org/wp-content/uploads/2018/08/NYULawReview-88-5-Fields.pdf

Gigerenzer, G. (2003). *Reckoning with risk: learning to live with uncertainty*. Penguin UK.Meester.

Guerra-Pujol, F. E. (2011). A Bayesian model of the litigation game. *European Journal of Legal Studies, 4(2): 220-240. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1891918*

Hunt, I., & Mostyn, J. (2020). Probability reasoning in judicial fact-finding. *The international Journal of evidence & Proof*, *24*(1):75-94. https://doi.org/10.1177/1365712719875753

Jaunzemis, A. D., Holzinger, M. J., Chan, M. W., & Shenoy, P. P. (2019). Evidence gathering for hypothesis resolution using judicial evidential reasoning. *Information Fusion*, *49*:26-45. https://doi.org/10.1016/j.inffus.2018.09.01

Jensen, F. V., & Nielsen, T. D. (2007). *Bayesian networks and decision graphs* (Vol. 2). New York: Springer.

Keppens, J. (2011, June). On extracting arguments from Bayesian network representations of evidential reasoning. In *Proceedings of the 13th international conference on artificial intelligence and law* (pp. 141-150). https://dl.acm.org/doi/10.1145/2018358.2018380

Neil, M., Fenton, N., & Nielson, L. (2000). Building large-scale Bayesian networks. *The Knowledge Engineering Review*, *15*(3): 257-284. https://www.eecs.qmul.ac.uk/~norman/papers/building_large_scale_bbns.pdf

Nulty & Ors versus Milton Keynes Borough Council [2011] EWHC 2847 (TCC) https://www.bailii.org/ew/cases/EWHC/TCC/2011/2847.html

Press, S. J. (2009). *Subjective and objective Bayesian statistics: Principles, models, and applications*. John Wiley & Sons.

R v Adams [1996] EWCA Crim 10 and 222.

R v T (2009). EWCA Crim 2439.

R., Collins, M., Gill, R., & Van Lambalgen, M. (2006). On the (ab) use of statistics in the legal case against the nurse Lucia de B. *Law, Probability & Risk*, *5*(3-4):233-250. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1312221

Sun, S., Zhang, C., & Yu, G. (2006). A Bayesian network approach to traffic flow forecasting. *IEEE Transactions on intelligent transportation systems*, *7*(1), 124-132. https://www.semanticscholar.org/paper/A-bayesian-network-approach-to-traffic-flow-Sun-Zhang/dd945f37dd0e2cf5443605ad4047cf9ad774eb1e

Tribe, L. H. (1970). Trial by mathematics: Precision and ritual in the legal process. *Harward Law Review.*, 84(6): 1329-1393. https://repository.library.georgetown.edu/handle/10822/763743

Verheij, B. (2014). To catch a thief with and without numbers: arguments, scenarios and probabilities in evidential reasoning. *Law, Probability and Risk*, *13*(3-4): 307-325. https://doi.org/10.1093/lpr/mgu011

Walker, V. R. (2007). Discovering the logic of legal reasoning. *Hofstra L. Rev.*, *35*:1687-1707 https://www.semanticscholar.org/paper/Discovering-the-Logic-of-Legal-Reasoning-Walker/92cfa33c1529c325c75fa1922544aebddf55d605