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COEFFICIENT OF VARIATION CONTROL CHART BASED ON CONDITIONAL EXPECTED VALUES FOR THE MONITORING OF CENSORED RAYLEIGH LIFETIMES

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ABSTRACT
This article deals with the monitoring of type-I censored data using coefficient of variation (CV) control chart based on conditional expected values (CEVs) for Rayleigh lifetimes under type-I censoring. In particular, the censored data is replaced by the CEV to develop an efficient design structure. The main focus is to detect shifts in the mean of Rayleigh lifetimes assuming censored data. The performance of the proposed CEV based CV chart is evaluated by the average run length (ARL). Besides the simulation study, monitoring of a real-life dataset of 30 average daily wind speeds (in kilometers/hour) for the month of November 2007 at Elanora Heights is also discussed.
Keywords: CEV, CV, type I censored, ARL, Average Run Length (ARL); Control Charts; Conditional Expected Values; type-I Censoring.

1. INTRODUCTION

It is difficult and fascinating work to find assignable causes in the monitoring of lifetime data, especially in medical and industrial research. However, time and expense constraints result in censored data, which leads to inadequate failure-time information. The traditional control charts, such as Shewhart charts, have poor performance in monitoring such experiments for possible special causes (Steiner and Mackay, 2000). In fact, these charts rarely respond in a reasonable timeframe, and as a result, traditional charts have little discriminatory monitoring power. Steiner and Mackay (2000) presented a one-side chart employing the conditional expected value (CEV) to address the undesirable features of the monitoring systems for censored data. The authors performed an empirical research to demonstrate the efficiency of the proposed method for monitoring highly censored data.

Different censoring systems have been introduced in the literature, including the frequently used type-I, type-II, interval, progressive, etc., (Lawless, 2002). The type-I censoring is the most often used scheme in business (Li and Kong, 2015). The lifespan of units in the interval [0, T] are observed for a fixed T, whereas observations with a lifetime larger than T are labeled as censored observations. In other words, it is impossible to determine the exact failure time of an observation greater than T. Lawless (2002) and Li and Kong (2015) provided a comprehensive detail about the types of censoring mechanism in statistics.

Abd-Elfattah et al. (2006a, 2006b) discussed Type II censored data. Steiner and Mackay (2000) developed a *CEV* control chart assuming a normal distribution. They used the transformation

$$W_{ij} = \begin{cases} t & \text{if } t \le C\,(\text{not censored}) \\ Wc & \text{if } t > C\,(\text{censored}) \end{cases}$$

to transform the data. Raza et al. (2021) developed a cumulative sum (CUSUM) control chart for Weibull lifetimes under Type 1 censoring. They used CEV and conditional mean (CM) approaches to

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design effective CUSUM design for censored data. They used average run length (ARL) to evaluate these charts. These ARL of CEV and CM based EWMA control charts are also compared to evaluate the efficiency of CUSUM over the EWMA. The final results showed that the CUSUM chart by using CEV and CM outperforms the EWMA charts. It also saw that CM approach based charts performed more effectively than the CEV based charts.

Raza and Siddiqi (2017) introduce the double exponential weighted moving average (DEWMA) control chart and further comparison with the EWMA control chart using type 1 censoring for Poisson-exponential distribution.

Due to the practical significance of censored data in different fields, numerous studies proposed efficient monitoring strategies. For example, Steiner and Mackay (2001a), Steiner and Mackay (2000), Steiner and Mackay (2001b), Zhang and Chen (2004), Lu and Tsai (2008), and Tsai and Lin (2009), Raza et al. (2015, 2016), Zhang et al. (2016), Raza and Siddique (2017) proposed control charts for monitoring censored data. Using a simulation study, the authors showed the superiority over the CEV charts. Azam et al. (2015) proposed a hybrid EWMA control chart using repetitive sampling. Apart from type-I censoring, type-II censored data monitoring schemes have also been introduced in the literature (Guo and Wang, 2014; Huang et al., 2017; Pascual and Li, 2012), however, this study deals with monitoring of type-I censored data, which is a commonly used censoring scheme in the industry. Castagliola et al. (2016) presented an excellent method to monitor the coefficient of variation (CV) by using the run rule (RR) type charts. If the mean and standard deviation of the process are not constant, then monitoring of CV is a profitable approach in statistical process control (SPC).

Zhang et al. (2018) suggested an innovative EWMA graph for the monitoring of CV. Raza et al. (2018) presented DEWMA charts based on CEV and CM to monitor the type 1 censored data. Their main motivation was the monitoring of mean level shift of Rayleigh distribution. However, they also discussed the relation between variance level shift and mean level shift for Rayleigh distribution. Ali et al. (2021) developed a conditional expected value hybrid double exponentially weighted moving average (CEVHDEWMA) chart for monitoring the location parameter of Weibull distribution using type 1 censored data.

Raza et al. (2016) explored the performance of EWMA control charts with Type I censoring for Poisson-exponential distribution. They used two types of sampling techniques, i.e., simple random sampling and rank set sampling.

Guo and Wang (2014) developed an innovative statistic to examine the shape parameter of Weibull distribution to deal with Type II censored data. In particular, they developed one-sided and two-sided ARL unbiased control charts, where the sample size, false alarm rate, and failure number determined the control limit of these charts. The performance of the chart was compared with existing range-based charts (Pascual and Li, 2012) by using Monte Carlo simulations. It is shown that the proposed chart gives better results as compared to the existing one. Further, parameter estimation effect is also discussed in the same study.

Huang et al. (2017) examined the control chart for exponential distribution under Type II censoring. They developed an average time to signal-unbiased control chart when the parameter is known.

Pascual and Li (2012) suggested a new control chart for observing the shape parameter of Weibull distribution with type II censored data. The chart was developed by using sample ranges of extreme small value distribution.

Haridy et al. (2014) suggested an attribute chart for attribute inspection plans to monitor the location and dispersion. Menzefricke (2010) proposed control charts for variance of a normal distribution and the CV of a log-normal distribution. To account for parameter uncertainty, a Bayesian methodology is used, and the control limits are derived from the variance's predictive distribution.

Variability monitoring is an essential aspect of advanced statistical process management. The traditional Shewhart R and S charts are used in situations where the variation of the in-control process readings is constant. However, in some cases, the coefficient of variance should be constant rather than the variance. In clinical chemistry, for example, this situation is commonly noticed, and hence R and S charts cannot be applied. Therefore, a chart for checking the coefficient of variation using balanced groups of observations that is similar to the S chart is proposed by Kang et al. (2007).

It is hard to track the procedure by means of conventional control charts while the production run is small and process parameters alter very often. The CV is extremely useful in this situation for monitoring process variability. The CV control chart is a useful method for controlling both the mean and variability of a process at the same time. The CV chart, in contrast, is insensitive to minor changes in CV magnitude. Hong et al. (2008) proposed a CV-EWMA control chart which is effective in detecting a small CV change. The CV-EWMA control chart scheme is very efficient for minor changes in mean and uncertainty of the method since it is a weighted average of both past and present CV values. They proposed design parameters and studied the performance of the CV-EWMA control chart using the ARL. On comparing the CV-EWMA control chart to the CV control chart, they discovered that the CV-EWMA control chart provides significantly larger in-control ARL and significantly shorter out-of-control ARL.

Many researchers have proposed CV based control charts in the literature like, McCracken and Chakraborti (2013), Miller and Karson (1970), Ahn (1995), Gong and Li (1999), Kang et al. (2007), Hong et al. (2008), Castagliola et al. (2011), You et al. (2016), Castagliola et al. (2011), Calzada and Scariano (2013), Castagliola et al. (2013), Castagliola et al. (2015), Amdouni et al. (2015).

The focus of this study is to introduce CEV based CV control charts for the monitoring of Type I censored data, which have not been discussed in the literature. For this, we consider type-I censored data from Rayleigh distribution and focus on its mean level. We also include a comparison of the proposed CEV and traditional CV charts using the ARL.

The rest of the study is organized as follows: Section 2 presents the general methodology of the proposed CEV based CV control chart. The performance of the proposed CEV based CV chart under different censoring rates is discussed in Section 3. Section 4 presents a dataset on the response time of an electric sockets experiment to illustrate the proposed methodology. Final remarks are given in Section 5.

2. THE PROPOSED CEV BASED CV CONTROL CHART

This section describes the structure of the suggested CEV based CV control chart for type-I censored data monitoring. Assume that the lifetimes, say L, follow the Rayleigh distribution with a parameter σ. The pdf of the Rayleigh distribution is

$$f(l,\sigma)=\frac{1}{\sigma^2}e^{-l^2/2\sigma^2}$$

where σ represents the scale parameter. Because of time or cost limitations, we record the times which are lower or equal to the predefined censoring time C. For the calculation of censoring time C, the censoring rate should be known (Ali et al., 2021).

$$P_c=1-F(\mathrm{l};\sigma)$$

where $F(\mathrm{l};\sigma)=1-\exp(-l^2/(2\sigma^2))$ represents the cumulative density function of the Rayleigh distribution. Since the lifetimes which are greater than censoring time will not be examined, typical monitoring structure results in delayed out-of-control signal. That is why, in such cases, it is appropriate to replace these values by the CEV. The CEV for Rayleigh distribution is computed by using the following expression

$$CEV(\mathrm{L})=E(\mathrm{L}\mid\mathrm{L}>\mathrm{c})=\frac{\alpha_o\Gamma(3/2,z_c)}{\exp(-z_c)}$$

where $\Gamma(a,x)=\int_a^\infty y^{a-1}\exp(-y)dy$ will be the incomplete upper gamma function $z_c=(c/\alpha)^2,\alpha>0$, c is the prefixed censoring time, $\alpha=(\sqrt{2})(\sigma)$ and α_o is the pre-specified value of α. (Raza et al., 2018)

The ARL is the most widely practiced performance assessment criterion. The ARL computed from the in-control data is known as the in-control ARL and denoted by ARL_o, while the ARL calculated from *Phase-II* data, that is, from a shifted process, is known as the out-of-control ARL and denoted by ARL_1. In general, the ARL is the average number of points plotted on a chart before a special cause signal is raised. Generally, ARL_o is fixed large while a chart with a smaller ARL_1 is said to be more efficient than the chart having larger ARL_1. For assessing the performance of the *CEV* bases CV charts, we fixed the desired ARL_o and find the corresponding UCL values, for the given δ and n. The steps to determine the control limits and ARLs for the pre-fixed values of P_c, m_o, n and ARL_o are given below:

1. Fix P_c and subgroup size.
2. Substitute the censored observations with the CEV, i.e., convert the data into $D_{ij(CEV)}$ and compute the CEV based CV statistic for different subgroups. Then, calculate the $(1-p)-$th quantile point, where p is a pre-specified false alarm probability.
3. Repeat the above step *L-times* (e.g., 100000 times) and compute the average to have the UCLs.
4. Continue to plot the value of the CEV based CV charting statistic against the subgroup numbers until the test statistic crosses the control limit. Record the corresponding subgroup number at which the first out-of-control signal appears.
5. Repeat Step 4, say M times (e.g., 100000 times), and compute the average, which is the ARL_0.

To calculate the out-of-control ARL, generate data from the Rayleigh distribution with the shifted parameter and check it against the monitoring threshold in Step 3. Store the subgroup numbers at which the monitoring statistic first falls beyond the control limit. Repeat this step many times and compute the average, which is ARL_1. The ARL_1 computation in this study is based on the steady state method.

Next, we discuss the efficiency of CEV based CV charts and compare its results with the traditional CV chart.

3. PERFORMANCE EVALUATIONS

The efficiency of the CEV based CV chart is discussed in this section. Besides this, a comparison of the CEV based CV chart and traditional CV chart is also given in this section. To investigate the efficiency of the charts, a simulation approach is used to calculate the ARL.

Table 1: Censoring Time C

	Σ		
P_c	**1.5**	**2.5**	**3.5**
0.2	2.19	2.83	3.35
0.3	1.90	2.45	2.90
0.4	1.65	2.14	2.53

Table 1 shows the censoring times calculated for different censoring rates, i.e., 0.2, 0.3 and 0.4. These censoring times are calculated using three different values of scale parameters which are 1.5, 2.5 and 3.5. We can see that by increasing censoring rate, the value of censoring times decreased whereas the censoring time also increased by increasing the value of scale parameter.

Table 2: UCL_{CEV} Values when n=3; σ = 1.5:

P_C	**σ = 1.5**		
	ARLo = 200	ARLo = 370	ARLo = 500
0.2	158.7217	158.8198	159.4753
0.3	143.3092	144.2514	143.5396
0.4	119.7857	122.951	124.6525

Table 2 shows the UCL values for CEV based CV control charts with σ=1.5 and n=3. The values of UCL are computed for 0.2, 0.3 and 0.4 censoring rates by assuming in-control ARL 200, 370 and 500, respectively.

Table 3: UCL_{CEV} Values when n=3; σ = 2.5:

P_C	**σ = 2.5**		
	ARLo = 200	ARLo = 370	ARLo = 500
0.2	162.2286	162.5448	162.0315
0.3	147.7835	147.7532	147.9631
0.4	130.3063	136.8307	132.1736

Table 3 shows the UCL values of the CEV based CV control chart with σ=2.5, n=3, ARL_0=200, 370, 500, and censoring rates=0.2, 0.3 and 0.4, respectively.

Table 4: UCL_{CEV} Values when n=3; σ = 3.5:

P_C	σ = 3.5		
	ARL_0 = 200	ARL_0 = 370	ARL_0 = 500
0.2	163.7405	164.0813	163.9705
0.3	150.9339	151.7135	151.3703
0.4	135.7315	137.4746	135.9653

Table 4 shows the UCL values for the CEV based CV control chart with σ=3.5, n=3, ARL_0=200, 370, 500 and censoring rates 0.2, 0.3, 0.4.

Table 5: UCL_{CEV} Values when n=5; σ = 1.5:

P_C	σ = 1.5		
	ARL_0 = 200	ARL_0 = 370	ARL_0 = 500
0.2	174.8182	160.4329	189.1093
0.3	158.3872	164.7226	166.1129
0.4	125.7739	130.9295	129.3867

Table 5 shows the UCL values for the CEV based CV control chart with σ=1.5, n=5, ARL_0=200, 370, 500 and censoring rates 0.2, 0.3, 0.4.

Table 6: UCL_{CEV} Values when n=5; σ= 2.5:

P_C	σ = 2.5		
	ARL_0 = 200	ARL_0 = 370	ARL_0 = 500
0.2	158.7179	177.0539	196.8322
0.3	171.888	166.2709	168.5107
0.4	150.973	142.8855	143.0361

Table 6 shows the UCL values for the CEV based CV control chart with σ=2.5, n=3, ARL_0=200, 370, 500 and censoring rates 0.2, 0.3, 0.4.

Table 7: UCL_{CEV} Values when n=5; σ= 3.5:

P_C	σ = 3.5		
	ARL_0 = 200	ARL_0 = 370	ARL_0 = 500
0.2	189.2469	186.5229	182.8413
0.3	177.2523	175.2934	178.123
0.4	148.929	155.2553	152.1368

Table 7 shows the UCL values for Table 2(c) shows the UCL values for the CEV based CV control chart with σ=3.5, n=3, ARL_0=200, 370, 500 and censoring rates 0.2, 0.3, 0.4.

TABLE 8: ARL_1 values at different censoring rates for CEV-CV control chart for n=3 with specified parameter σ = 1.5 using ARL_0= 200, 370 and 500.

P_C	0.2			0.3			0.4		
Shifts	**200**	**370**	**500**	**200**	**370**	**500**	**200**	**370**	**500**
5% Decrease	51.340	54.782	56.072	29.057	54.782	31.800	13.785	14.300	14.402
10% Decrease	50.667	48.999	53.799	28.047	48.999	30.174	13.147	13.427	14.087
15% Decrease	44.555	47.539	51.643	27.496	47.539	29.292	12.942	12.969	13.689

20% Decrease	43.850	46.210	48.599	25.015	46.210	27.568	12.249	12.914	13.473
25% Decrease	40.553	44.579	46.554	23.860	44.579	26.480	11.967	12.730	13.229
30% Decrease	39.524	42.843	43.846	22.460	42.843	25.013	11.311	12.587	12.943
35% Decrease	38.199	40.102	42.817	20.151	40.102	24.909	10.850	11.988	12.189
40% Decrease	31.261	33.928	34.949	19.915	33.928	22.620	10.793	10.668	11.453

Table 8 shows the values of the out-of-control ARL_1 calculated for different censoring rates when n=3 with parameter $\sigma = 1.5$ with in-control ARL_0 fixed at 200, 370 and 500. For ARL_0=200 and at 0.2 censoring rate with 5% shift, the control chart gives an ARL_1 = 51.340. If there is a 10% decrease shift in the parameter, the ARL_1 reduces to 50.667. As the value of shift increases, the value of ARL_1 decreases. If there is a 40% decreasing shift in the parameter, the ARL_1 is 31.261. For censoring rate 0.3, the ARL_1 is 29.057 for 5% shift and the smallest ARL_1 is 19.915 which is obtained at 40% shift in the process. For censoring rate 0.4, the ARL_1 is observed as 13.785 with 5% shift and the ARL_1=10.793 for 40% shift. For the case of 370 in-control ARL at censoring rate 0.2, the value of ARL_1 is 54.782 when 5% shift is introduced and ARL_1 is 33.928 when 40% shift is introduced. For the censoring rate 0.3, the value of ARL_1 is 54.782 when 5% shift and ARL1=33.928 with 40% shift. For censoring rate 0.4, the value of ARL_1 is 14.300 with 5% shift and ARL_1 is 10.668 when 40% shift is introduced. For the case of ARL_0=500 at censoring rate 0.2, the value of ARL_1 is 56.072 when 5% shift is introduced and ARL_1=34.949 when 40% shift is introduced. For the censoring rate 0.3, the value of ARL_1 is 31.800 when 5% shift is introduced and ARL_1 is 22.620 when 40% shift is introduced. Similarly, for censoring rate 0.4, the value of ARL_1 is 14.402 when 5% shift is introduced and ARL_1 is 11.453 when 40% shift is introduced. The results show that the CEV-CV control chart is efficient for detecting the shifts more rapidly when the censoring rate is 0.4.

TABLE 9: ARL_1 values at different censoring rates for CEV-CV control chart for n=3 with specified parameter $\sigma = 2.5$ using ARL_0= 200, 370 and 500.

P_C	0.2			0.3			0.4		
Shifts	**200**	**370**	**500**	**200**	**370**	**500**	**200**	**370**	**500**
5% Decrease	53.317	57.877	55.817	30.230	30.520	31.110	14.491	14.212	14.645
10% Decrease	49.099	53.766	54.631	26.092	28.176	30.069	13.610	13.926	13.980
15% Decrease	48.981	51.750	53.599	25.770	27.970	29.207	13.331	13.695	13.721
20% Decrease	46.850	49.490	52.171	25.132	26.546	28.940	12.398	13.086	13.213
25% Decrease	42.883	46.098	46.785	24.040	25.302	26.489	11.973	12.269	12.737
30% Decrease	38.741	43.475	43.463	22.366	23.752	25.579	11.512	12.184	12.397
35% Decrease	38.604	42.419	40.649	20.374	22.726	23.561	11.387	11.538	11.752
40% Decrease	29.187	31.746	34.646	19.674	20.165	21.358	10.898	10.804	11.472

Table 9 shows the values of ARL_1 calculated for different censoring rates, n=3 and $\sigma = 2.5$. As the shift in the scale parameter increased, the value of ARL_1 decreased which shows that the control chart detects out-of-control state more rapidly. The efficiency of the control chart increased as the censoring rate increased. At 40% censoring rate, the ARL_1 values are considerably smaller as compared to 20% and 30% censoring rate.

TABLE 10: ARL_1 values at different censoring rates for CEV-CV control chart for n=3 with specified parameter σ = 3.5 using ARL_0= 200, 370 and 500.

P_C	0.2			0.3			0.4		
Shifts	**200**	**370**	**500**	**200**	**370**	**500**	**200**	**370**	**500**
5% Decrease	53.819	55.070	57.428	29.552	29.311	30.720	13.935	14.617	14.341
10% Decrease	49.144	53.039	56.333	26.645	29.018	29.146	13.786	14.063	13.936
15% Decrease	48.474	49.538	51.145	25.778	28.299	28.123	12.761	13.582	13.771
20% Decrease	47.765	48.426	50.226	25.061	25.559	26.837	12.615	13.031	13.194
25% Decrease	46.578	48.248	49.493	22.788	24.910	25.610	12.323	12.887	13.009
30% Decrease	42.126	41.930	47.069	21.636	24.567	24.700	11.850	12.453	12.729
35% Decrease	34.519	40.341	39.733	20.657	21.546	23.431	11.549	12.119	12.251
40% Decrease	30.856	33.789	35.522	18.868	19.937	22.356	10.726	11.023	11.937

Table 10 shows ARL_1 for different censoring rates, n=3, σ = 3.5, and assuming in-control ARL_0=200, 370 and 500. As the value of shift decreases, the value of ARL_1 also decreases which shows that the control chart detects out-of-control state more rapidly. The efficiency of the control chart increased as the censoring rate increased. At a 40% censoring rate, the ARL_1 values are considerably smaller as compared to 20% and 30% censoring rates.

TABLE 11: ARL_1 values at different censoring rates for CEV-CV control chart for n=5 with specified parameter σ = 1.5 using ARL_0= 200, 370 and 500.

P_C	0.2			0.3			0.4		
Shifts	**200**	**370**	**500**	**200**	**370**	**500**	**200**	**370**	**500**
5% Decrease	47.762	49.604	52.917	54.031	55.464	56.394	47.328	51.070	51.752
10% Decrease	45.947	49.217	51.149	52.331	54.149	54.534	45.673	47.201	47.267
15% Decrease	44.834	47.264	49.131	45.382	47.291	50.917	38.875	42.228	42.754
20% Decrease	43.400	46.447	48.605	40.533	43.992	48.421	34.329	38.356	39.602
25% Decrease	34.657	39.712	42.945	36.731	38.198	42.492	31.632	32.912	34.447
30% Decrease	31.128	35.799	35.604	29.351	31.898	36.432	24.970	27.579	29.210
35% Decrease	22.087	24.561	27.034	22.483	24.702	26.889	20.572	22.157	23.029
40% Decrease	18.591	18.608	19.785	17.217	17.861	18.544	17.285	17.582	18.669

Table 11 shows the values of ARL_1 calculated for different censoring rates, n=5, σ = 1.5, and ARL_0=200, 370 and 500, respectively. As the shift decreases, the ARL_1 also decreases. Also, the ARL_1 values are slightly smaller with ARL_0=200 as compared to 370 and 500 in-control ARL_0.

TABLE 12: ARL_1 values at different censoring rates for CEV-CV control chart for n=5 with specified parameter σ = 2.5 using ARL_0= 200, 370 and 500.

P_C	0.2			0.3			0.4		
Shifts	**200**	**370**	**500**	**200**	**370**	**500**	**200**	**370**	**500**
5% Decrease	51.740	53.465	53.448	53.101	56.697	56.039	47.245	50.981	51.963
10% Decrease	51.315	50.866	51.438	49.709	52.322	53.678	44.979	47.254	48.231
15% Decrease	46.234	49.738	50.454	47.091	51.218	52.334	39.630	42.099	43.915
20% Decrease	43.613	46.179	49.814	43.656	45.534	48.775	35.606	36.508	38.430
25% Decrease	38.368	40.240	44.463	35.145	37.318	41.648	29.290	32.136	36.280
30% Decrease	30.204	33.172	35.066	30.419	33.419	35.824	26.389	27.153	30.715
35% Decrease	25.285	26.175	27.966	23.841	25.598	27.261	20.331	22.789	24.278
40% Decrease	17.554	17.658	19.216	17.933	20.461	21.565	16.124	17.624	19.109

Table 12 lists the ARL_1 values calculated with different censoring rates, n=5, and **$\sigma = 2.5$.** The results show that the CEV-CV control chart detects out-of-control shifts more quickly when the censoring rate is 0.4 as compared to censoring rate 0.2 and 0.3. Table 9 shows the values of the out-of-control ARL_1 for n=5 and **$\sigma = 3.5$.** It is noticed that as the value of shift decreases, the value of ARL_1 also decreases which shows that the control chart detects out-of-control state more rapidly. If ARL_0 is fixed at 200, the ARL_1 values are slightly smaller as compared to 370 and 500 ARL_0. This shows that the CEV-CV control chart detects out-of-shift more quickly when the censoring rate is 0.4 as compared to censoring rates 0.2 and 0.3, respectively.

TABLE 9: ARL_1 values at different censoring rates for CEV-CV control chart for n=5 with specified parameter $\sigma = 3.5$ using ARL_0= 200, 370 and 500.

P_C	0.2			0.3			0.4		
Shifts	**200**	**370**	**500**	**200**	**370**	**500**	**200**	**370**	**500**
5% Decrease	49.286	52.563	52.326	55.840	55.017	56.792	48.775	48.934	52.399
10% Decrease	48.848	51.057	47.227	48.861	53.543	55.174	45.364	45.372	50.824
15% Decrease	46.567	50.815	46.756	47.489	46.940	52.408	38.123	43.164	43.988
20% Decrease	42.384	47.345	45.838	43.870	45.872	47.426	35.806	38.959	40.955
25% Decrease	38.787	41.826	42.248	39.684	38.356	43.504	30.398	33.162	35.177
30% Decrease	32.991	34.669	35.805	32.814	35.424	37.512	21.711	28.440	29.578
35% Decrease	25.098	27.329	28.581	25.344	26.115	29.537	18.518	24.124	24.679
40% Decrease	17.625	19.438	21.521	19.610	19.823	20.242	17.640	18.140	19.353

3.1 ARL CURVES

To compare the performance of the proposed CEV-CV control chart and simple CV control chart we made the graphs of ARL curves.

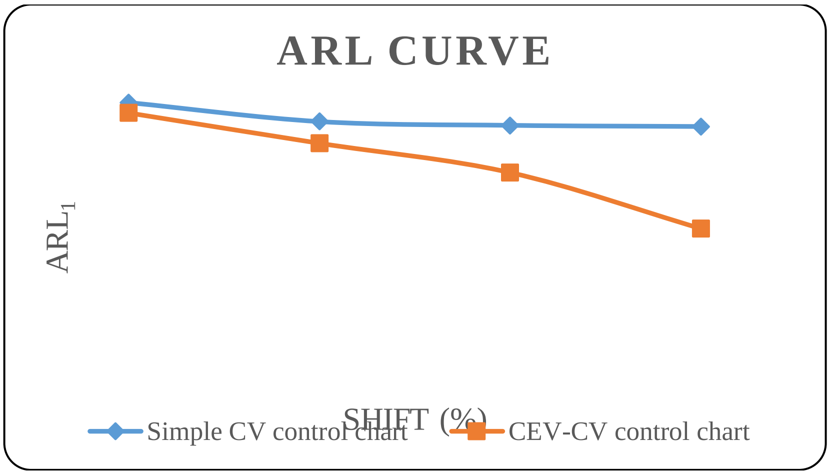

Figure 1: ARL curve when n=3; P_c=0.2; σ=1.5

Figure 1 depicts the ARL curves for a simple CV control chart and CEV-CV control chart assuming n=3; P_c=0.2; σ=1.5. One can see from the figure that the ARL_1 value of the CEV-CV control chart is smaller than the simple CV control chart. Thus, the CEV-CV control chart outperforms the simple CV control chart.

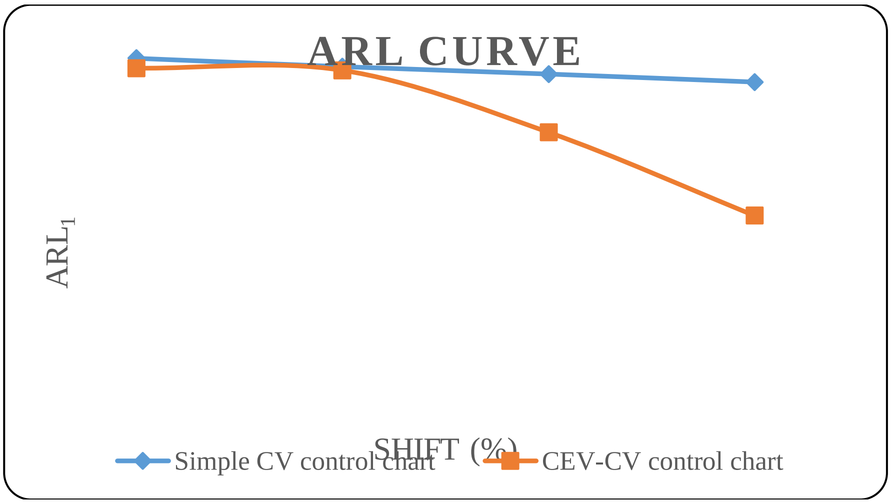

Figure 2: ARL curve when n=3; P_c=0.2; σ=2.5

Figure 2 presents the ARL curves for a simple CV control chart and CEV-CV control chart with n=3; P_c=0.2; σ=2.5. Since the ARL_1 values of the CEV-CV control chart are smaller than the simple CV control chart, the CEV-CV control chart outperforms.

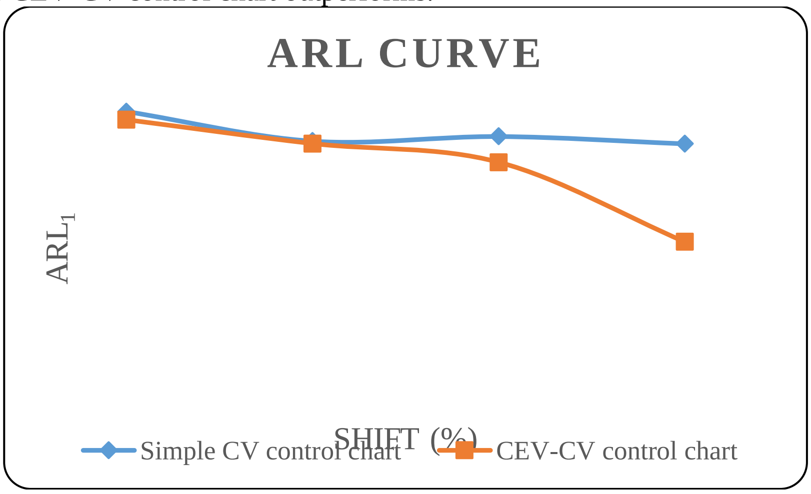

Figure 3: ARL curve when n=3; P_c=0.2; σ=3.5

Figure 3 presents the ARL curves for simple CV and CEV-CV control charts with n=3; P_c=0.2; σ=3.5. Since the ARL_1 values of the CEV-CV control chart are smaller than the simple CV control chart, the CEV-CV control chart outperforms the simple CV control chart.

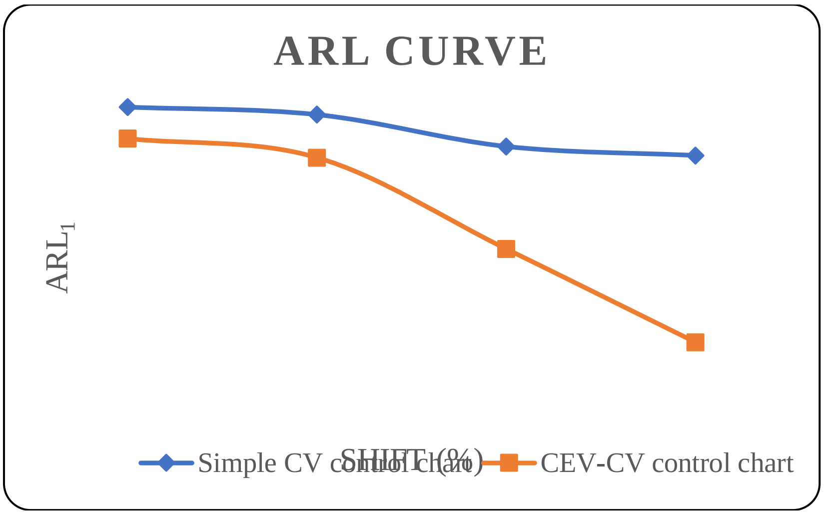

Figure 4: ARL curve when n=5; P_c=0.2; σ=1.5

Figure 4 shows the ARLs of CV control chart and CEV-CV control chart by assuming n=5; P_c=0.2; σ=1.5. Again the CEV-CV control chart outperforms the simple CV control chart.

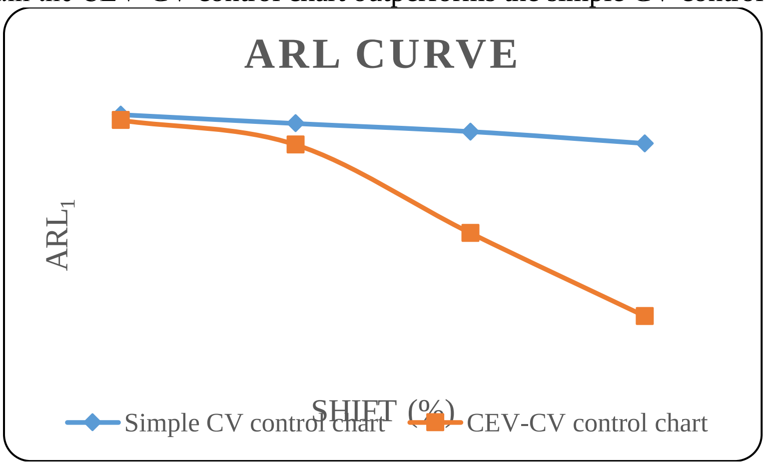

Figure 5: ARL curve when n=5; P_c=0.2; σ=2.5

Figure 5 displays the ARLs of ordinary CV and CEV-CV control charts with n=5; P_c=0.2; σ=2.5. It is concluded that the CEV-CV control chart outperforms the simple CV control chart.

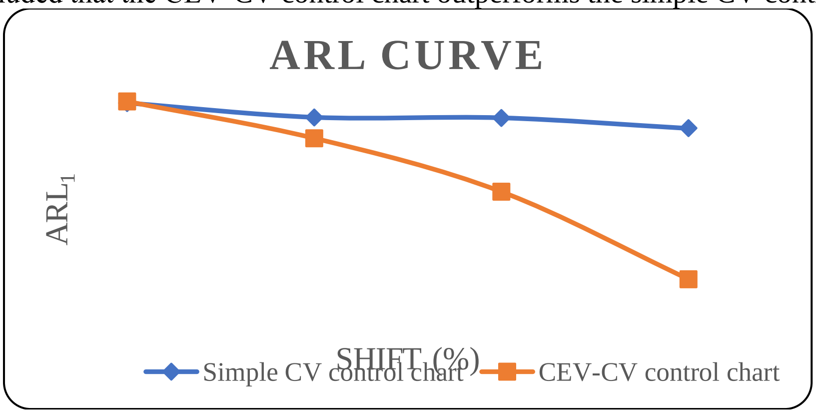

Figure 6: ARL curve when n=5; P_c=0.2; σ=3.5

Figure 6 shows the ARLs of simple CV and CEV-CV control charts when n=5; P_c=0.2; σ=3.5, and CEV-CV control chart outperforms the simple CV control chart.

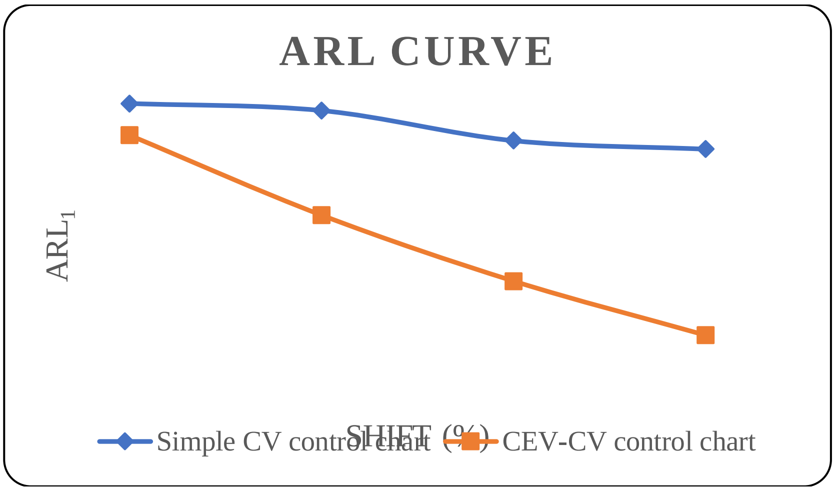

Figure 7: ARL curve when n=5; P_c=0.4; σ=1.5

Figure 7 depicts the ARL values of simple CV and CEV-CV control charts when n=5; P_c=0.4; σ=1.5 and it is evident that the ARL_1 values of CEV-CV control chart are smaller than the simple CV control chart.

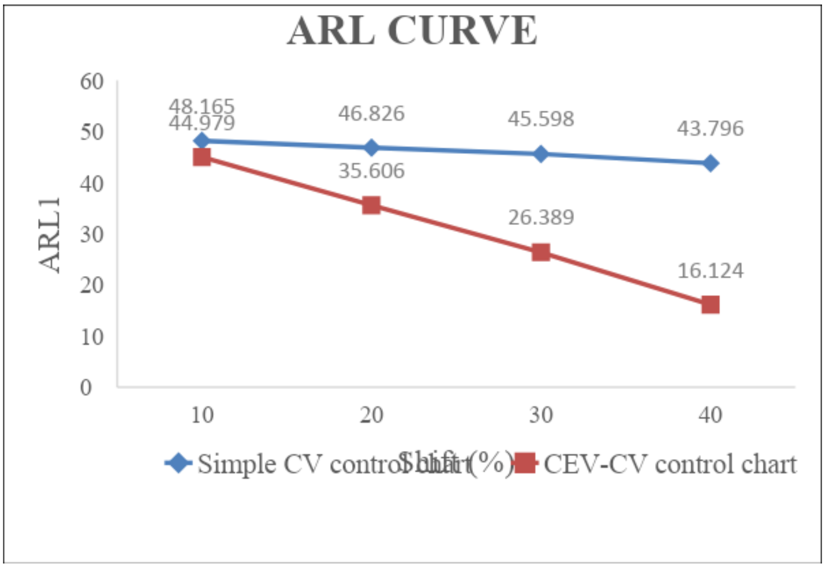

Figure 8: ARL curve when n=5; P_c=0.4; σ=2.5

Figure 8 presents a comparison of ARL_1 for a simple CV control chart and CEV-CV control chart with n=5; P_c=0.4; σ=2.5 and it is noticed that the CEV-CV control chart outperforms the simple CV control chart. The same conclusion is validated from Figure 9.

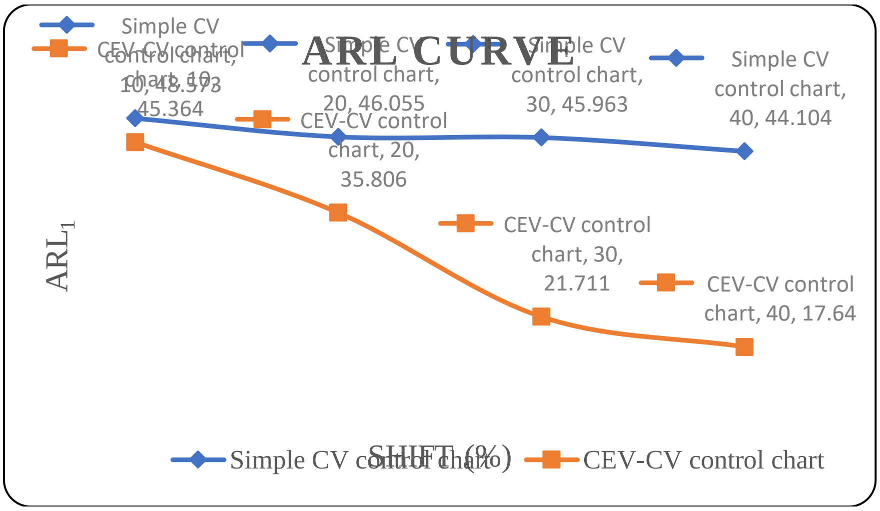

Figure 9: ARL curve when n=5; P_c=0.4; σ=3.5

In short, the important findings are given below:

i) For low censoring rate, an increasing shift in the scale parameter is more efficiently detected.

ii) The out-of-control is more efficiently detected with the increased shift in scale parameter.

iii) It is noticed that the CEV based CV chart is superior to the traditional CV chart. Furthermore, it is also found that the *ARL* follows the unbiasedness property, i.e., out-of-control ARL never exceeds the ARL_0 value.

iv) The CEV based CV chart outperforms the traditional CV chart.

4. Applications

"To study the application of the proposed CEV-CV control chart on real life data, we have taken a dataset of 30 average daily wind speeds (in kilometers/hour) for the month of November 2007 at Elanora Heights, a north-eastern suburb of Sydney, Australia; (Best et al. 2010). The data is given below

2.7	3.2	2.1	4.8	7.6	4.7	4.2	4.0	2.9	2.9
4.6	4.8	4.3	4.6	3.7	2.4	4.9	4.0	7.7	10.0
5.2	2.6	4.2	3.6	2.5	3.3	3.1	3.7	2.8	4.0

To check the distribution of the data, we use *easyfit* software and found that the data follows Rayleigh distribution with σ = 3.327.

We generate additional 30 random samples of size 3 from the dataset of daily wind speed with a shifted scale parameter. To be specific, we introduced a 20% decreasing shift in the scale parameter σ and calculated the ARL_1 which is equal to 7.475 (Figure 10). This value indicates that the control chart needs 7 sample points on average to detect an out-of-control signal with a 20% decreasing shift.

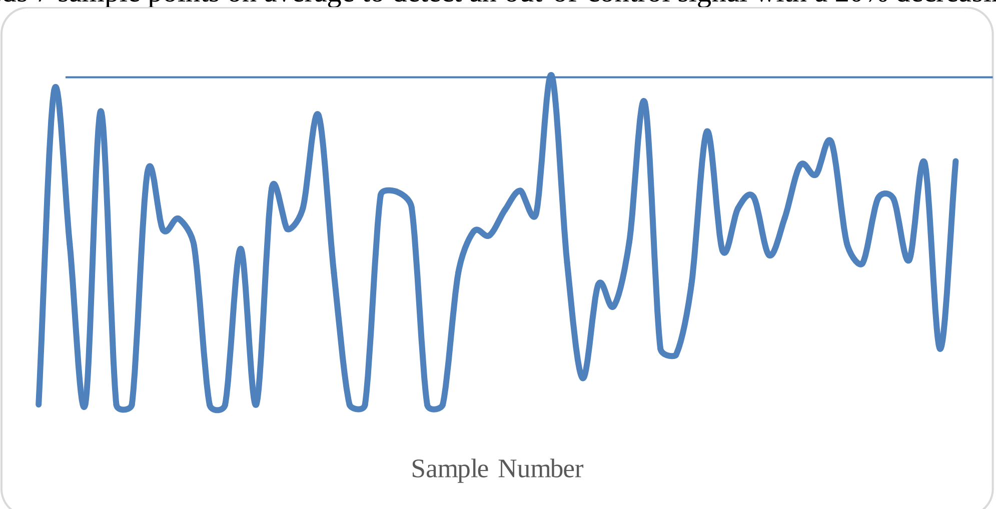

Figure 10: CEV-CV chart for Daily wind speed data "

5. CONCLUSION

"In this article, we proposed a new CEV based CV control chart for Type I censored data monitoring assuming Rayleigh distribution and evaluated the performance of these charts using a simulation study. We considered n=3 and n=5 with three different values of scale parameter of the distribution, i.e., 1.5, 2.5 and 3.5. Different censoring rates are used for the calculation of CEVs. It is found that as the magnitude of shift increased, the ARL_1 values decreased which means that the control chart detects out-of-control signals more quickly. These ARL_1 values are calculated for specified in-control ARL_0 which are 200, 370 and 500. For sample size 3, the CEV-CV control chart performs more efficiently with a high censoring rate, i.e., 40%, as compared to 20% and 30%. The efficiency of the control chart also increases with the increase in censoring rate. For both small and large shifts, the CEV-CV control chart with 40% censoring rate detect out-of-signal more quickly. For ARL_0=200, the chart gives the smallest ARL_1 as compared to 370 and 500 in-control ARL_0. As the sample size increased to 5, the control chart with 0.2 censoring rate detected the large shift more quickly as compared to sample size 3. For censoring rate 0.4, the CEV-CV control chart gives the smallest ARL_1 as compared to 0.2 and 0.3. The comparison of the proposed CEV-CV control chart to a simple CV control chart reveals that the former chart is more efficient for detecting a shift. Thus, the proposed CEV-CV control chart outperforms the simple CV control chart. To see the application of the proposed CEV-CV control chart, a real data example of 30 average daily wind speeds is also included in this paper. From the application, it is shown that the control chart needs 7 sample points to detect an out-of-control signal.

Declarations:

Funding: No funding received from any source to conduct this study.

Conflicts of interest/Competing interests. The authors declare no conflict of interest.

Availability of data and material: The data used in the article are available from the corresponding author. Also, the reference of real data is also cited in the text.

Code availability: An algorithm is mentioned to replicate the results.

Authors' contributions: All authors contributed equally.

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Appendix A: Samples and Calculations

RAYLIEGH			Dij			MEAN	SD	CV	UCL
1.471135	0.682294	3.363658	1.471135	0.682294	5.107	2.420143	2.360078	97.51811	129.0983
2.051321	2.089189	2.419881	2.051321	2.089189	2.419881	2.186797	0.202743	9.271216	129.0983
2.618439	1.871851	0.821133	5.107	1.871851	0.821133	2.599995	2.233788	85.9151	129.0983
1.480128	0.433819	3.902656	1.480128	0.433819	5.107	2.340316	2.452468	104.7922	129.0983
1.578465	2.43343	2.793356	1.578465	2.43343	5.107	3.039632	1.84072	60.55734	129.0983
0.548548	1.890562	2.625547	0.548548	1.890562	5.107	2.51537	2.342575	93.13044	129.0983
0.629757	0.80534	1.995672	0.629757	0.80534	1.995672	1.14359	0.743129	64.98215	129.0983
1.245913	1.718476	1.833706	1.245913	1.718476	1.833706	1.599365	0.311474	19.47483	129.0983
0.412014	0.790938	1.213011	0.412014	0.790938	1.213011	0.805321	0.400692	49.75556	129.0983
0.726176	0.177768	2.29566	0.412014	0.790938	1.213011	0.805321	0.400692	49.75556	129.0983
1.997629	2.187867	1.807191	1.997629	2.187867	1.807191	1.997562	0.190338	9.528521	129.0983
1.191937	1.924113	2.425166	1.191937	1.924113	2.425166	1.847072	0.620214	33.57821	129.0983
3.692983	1.586588	4.80378	5.107	1.586588	5.107	3.933529	2.032511	51.67143	129.0983
1.684446	2.117991	1.026157	1.684446	2.117991	1.026157	1.609531	0.549759	34.15646	129.0983
1.803541	3.191923	2.654436	1.803541	5.107	5.107	4.005847	1.907253	47.61173	129.0983
2.217185	1.364228	0.255681	2.217185	1.364228	0.255681	1.279031	0.983523	76.89597	129.0983
3.15838	1.882056	1.656595	5.107	1.882056	1.656595	2.881884	1.930302	66.98056	129.0983
2.520815	3.238732	2.363489	5.107	5.107	2.363489	4.192496	1.583967	37.78099	129.0983
2.967042	1.897054	0.80298	5.107	1.897054	0.80298	2.602345	2.237012	85.96141	129.0983
2.298446	0.468165	2.120194	2.298446	0.468165	2.120194	1.628935	1.0092	61.95456	129.0983
1.459319	0.880196	2.936515	1.459319	0.880196	5.107	2.482171	2.291536	92.31983	129.0983
1.078483	2.190758	3.635592	1.078483	2.190758	5.107	2.79208	2.080488	74.51389	129.0983
2.295042	3.768096	1.977212	2.295042	5.107	1.977212	3.126418	1.72258	55.09756	129.0983
1.693961	1.769089	1.289857	1.693961	1.769089	1.289857	1.584302	0.257749	16.26894	129.0983
2.579819	2.814422	2.260669	5.107	5.107	2.260669	4.158223	1.64333	39.52001	129.0983
1.659297	2.681053	1.91322	1.659297	5.107	1.91322	2.893172	1.92143	66.41258	129.0983
2.382953	1.76374	2.189582	2.382953	1.76374	2.189582	2.112092	0.316796	14.99915	129.0983
2.783719	0.68186	3.528656	5.107	0.68186	5.107	3.631953	2.554856	70.34385	129.0983
3.702232	4.652499	2.070534	5.107	5.107	2.070534	4.094845	1.753104	42.81247	129.0983
2.398368	1.564775	1.721805	2.398368	1.564775	1.721805	1.894983	0.442959	23.37534	129.0983
2.209012	1.556516	1.601136	2.209012	1.556516	1.601136	1.788888	0.364522	20.377	129.0983
0.412404	4.916563	1.521777	0.412404	5.107	1.521777	2.34706	2.453697	104.5434	129.0983
1.097086	1.222473	5.066057	1.097086	1.222473	5.107	2.475519	2.279791	92.09345	129.0983
1.621235	1.705312	1.121147	1.621235	1.705312	1.121147	1.482564	0.315807	21.30142	129.0983
3.21587	2.971659	0.493318	5.107	5.107	0.493318	3.569106	2.663711	74.63243	129.0983
0.714164	3.008044	2.402361	0.714164	5.107	2.402361	2.741175	2.215931	80.83871	129.0983
1.734271	0.794984	4.485471	1.734271	0.794984	5.107	2.545418	2.267563	89.0841	129.0983
2.33575	2.858507	2.21381	2.33575	5.107	2.21381	3.218853	1.636319	50.83548	129.0983
3.46655	0.503696	1.505475	5.107	0.503696	1.505475	2.372057	2.420914	102.0597	129.0983
2.617836	1.656037	1.590839	5.107	1.656037	1.590839	2.784625	2.0115	72.23591	129.0983
2.516016	1.958422	3.559974	5.107	1.958422	5.107	4.057474	1.817832	44.80207	129.0983
2.865441	1.105934	1.233389	5.107	1.105934	1.233389	2.482108	2.274117	91.62038	129.0983
1.357295	4.034043	1.493023	1.357295	5.107	1.493023	2.652439	2.126795	80.18261	129.0983

1.556248	1.219095	0.21314	1.556248	1.219095	0.21314	0.996161	0.698756	70.14485	129.0983
2.128116	1.724023	1.492534	2.128116	1.724023	1.492534	1.781558	0.321674	18.05574	129.0983
1.286818	2.961882	1.953234	1.286818	5.107	1.953234	2.782351	2.040594	73.34065	129.0983
4.314494	1.466767	3.143783	5.107	1.466767	5.107	3.893589	2.101689	53.9782	129.0983
0.364438	1.915257	2.141779	0.364438	1.915257	2.141779	1.473825	0.96741	65.63941	129.0983
2.961845	0.70421	2.696256	5.107	0.70421	5.107	3.639403	2.541952	69.84529	129.0983
2.450837	1.82329	2.259583	5.107	1.82329	2.259583	3.063291	1.783297	58.21505	129.0983
1.819483	1.257233	1.876204	1.819483	1.257233	1.876204	1.650973	0.342167	20.72516	129.0983
3.532945	1.695871	3.860049	5.107	1.695871	5.107	3.969957	1.969416	49.608	129.0983
2.136282	4.09641	1.424362	2.136282	5.107	1.424362	2.889215	1.953365	67.60886	129.0983
1.591475	3.597052	1.373674	1.591475	5.107	1.373674	2.690716	2.095395	77.87499	129.0983
1.341987	2.994698	2.161803	1.341987	5.107	2.161803	2.870263	1.979967	68.98205	129.0983
3.46082	2.33272	1.232247	5.107	2.33272	1.232247	2.890656	1.996721	69.07503	129.0983
2.645253	1.980547	3.755891	5.107	1.980547	5.107	4.064849	1.805059	44.40654	129.0983
2.057372	3.711336	1.476691	2.057372	5.107	1.476691	2.880354	1.950067	67.70233	129.0983
0.924725	2.569745	1.500975	0.924725	5.107	1.500975	2.5109	2.266675	90.27341	129.0983
1.474531	2.583619	1.440488	1.474531	5.107	1.440488	2.674006	2.107103	78.79948	129.0983
0.634185	1.298928	1.271059	0.634185	1.298928	1.271059	1.068057	0.376003	35.20439	129.0983
1.054286	2.27996	0.835342	1.054286	2.27996	0.835342	1.389863	0.778581	56.01857	129.0983
1.674931	4.397629	2.720744	1.674931	5.107	5.107	3.962977	1.981506	50.00044	129.0983
4.087779	1.229768	1.946521	5.107	1.229768	1.946521	2.761097	2.062979	74.71592	129.0983
2.27422	1.246049	2.698305	2.27422	1.246049	5.107	2.875756	1.99953	69.53057	129.0983
3.23614	3.220256	1.512682	5.107	5.107	1.512682	3.908894	2.07518	53.08868	129.0983
2.154742	3.718465	0.673583	2.154742	5.107	0.673583	2.645108	2.25702	85.32808	129.0983
2.823205	2.241614	2.595601	5.107	2.241614	5.107	4.151871	1.654331	39.84543	129.0983
2.068949	1.902517	1.179796	2.068949	1.902517	1.179796	1.717087	0.472691	27.52865	129.0983
2.025729	2.194842	1.314567	2.025729	2.194842	1.314567	1.845046	0.467125	25.31779	129.0983
1.435422	1.877363	1.57942	1.435422	1.877363	1.57942	1.630735	0.225395	13.82168	129.0983
0.347414	1.698581	1.509525	0.347414	1.698581	1.509525	1.185173	0.731653	61.73386	129.0983
0.998014	1.415097	0.979432	0.998014	1.415097	0.979432	1.130847	0.246342	21.78388	129.0983
1.267443	1.513569	2.174555	1.267443	1.513569	2.174555	1.651856	0.469101	28.3984	129.0983
2.450837	1.82329	2.259583	5.107	1.82329	2.259583	3.063291	1.783297	58.21505	129.0983
1.819483	1.257233	1.876204	1.819483	1.257233	1.876204	1.650973	0.342167	20.72516	129.0983
3.532945	1.695871	3.860049	5.107	1.695871	5.107	3.969957	1.969416	49.608	129.0983
2.136282	4.09641	1.424362	2.136282	5.107	1.424362	2.889215	1.953365	67.60886	129.0983
1.591475	3.597052	1.373674	1.591475	5.107	1.373674	2.690716	2.095395	77.87499	129.0983
1.341987	2.994698	2.161803	1.341987	5.107	2.161803	2.870263	1.979967	68.98205	129.0983
3.46082	2.33272	1.232247	5.107	2.33272	1.232247	2.890656	1.996721	69.07503	129.0983
2.645253	1.980547	3.755891	5.107	1.980547	5.107	4.064849	1.805059	44.40654	129.0983
2.057372	3.711336	1.476691	2.057372	5.107	1.476691	2.880354	1.950067	67.70233	129.0983
0.924725	2.569745	1.500975	0.924725	5.107	1.500975	2.5109	2.266675	90.27341	129.0983
1.474531	2.583619	1.440488	1.474531	5.107	1.440488	2.674006	2.107103	78.79948	129.0983
0.634185	1.298928	1.271059	0.634185	1.298928	1.271059	1.068057	0.376003	35.20439	129.0983
1.054286	2.27996	0.835342	1.054286	2.27996	0.835342	1.389863	0.778581	56.01857	129.0983
1.674931	4.397629	2.720744	1.674931	5.107	5.107	3.962977	1.981506	50.00044	129.0983

4.087779	1.229768	1.946521	5.107	1.229768	1.946521	2.761097	2.062979	74.71592	129.0983
2.27422	1.246049	2.698305	2.27422	1.246049	5.107	2.875756	1.99953	69.53057	129.0983
3.23614	3.220256	1.512682	5.107	5.107	1.512682	3.908894	2.07518	53.08868	129.0983
2.154742	3.718465	0.673583	2.154742	5.107	0.673583	2.645108	2.25702	85.32808	129.0983
2.823205	2.241614	2.595601	5.107	2.241614	5.107	4.151871	1.654331	39.84543	129.0983
2.068949	1.902517	1.179796	2.068949	1.902517	1.179796	1.717087	0.472691	27.52865	129.0983
2.025729	2.194842	1.314567	2.025729	2.194842	1.314567	1.845046	0.467125	25.31779	129.0983
1.435422	1.877363	1.57942	1.435422	1.877363	1.57942	1.630735	0.225395	13.82168	129.0983
1.170178	3.297372	0.663318	1.170178	5.107	0.663318	2.313499	2.432481	105.143	129.0983
1.343716	1.71792	1.813294	1.343716	1.71792	1.813294	1.624977	0.248203	15.27427	129.0983
1.740542	2.435828	3.065813	1.740542	2.435828	5.107	3.094457	1.777246	57.43322	129.0983
1.432226	1.646867	2.985715	1.432226	1.646867	5.107	2.728698	2.062464	75.58421	129.0983
1.580812	2.316836	1.38027	1.580812	2.316836	1.38027	1.759306	0.493137	28.0302	129.0983