

## HYBRID DOUBLE EXPONENTIALLY WEIGHTED MOVING AVERAGE (HDEWMA) CONTROL CHART FOR INVERSE RAYLEIGH DISTRIBUTION

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### ABSTRACT

*In this paper, we proposed a Hybrid Double Exponentially Weighted Moving Average HDEWMA control chart. The proposed control chart is based on Inverse Rayleigh Distributed lifetimes using simple random sampling (SRS) and ranked set sampling (RSS). Out-of-control-Average Run Length ( $ARL_1$ ) is used to evaluate the performance of the proposed control chart. The HDEWMA control chart is compared with traditional/simple EWMA and CUSUM control charts. The performance of the control chart is evaluated using out of control average run length ( $ARL_1$ ). A simulated example is used to compare the proposed HDEWMA, traditional/simple EWMA chart and CUSUM control chart. It is observed that the proposed HDEWMA control chart performs simple EWMA and CUSUM control charts. The HDEWMA control chart can be used for efficient monitoring of the production process in manufacturing industries where the data is coming from inverse Rayleigh Distribution.*

**Keywords:** Hybrid, EWMA, CUSUM, DEWMA, average run length (ARL).

### 1. INTRODUCTION

The most effectual method in SPC for perpetuate process is control charting. The idea of control charts was introduced by Shewhart (1931) proposed control chart has an assumption that the data should follow the Normal distribution. The 99.73% of observations that are sampled from a Normal distribution are included within 3 standard deviations of the mean, i.e. between the limits  $\mu \pm 3\sigma$ . Observations are said to be in-control if they remain within these control limits and the process goes out of control when the observations plots outside these limits.

The use of statistical quality control charts in many sectors of life revealed that nearly all of the control charts are meant to offer data on the quality characteristic/studied variable. If we can obtain some information about another variable(s) that is connected with our variable of interest, we may use the efficient charting statistic to improve the control chart's efficiency. The additional information is known as auxiliary information, and the variable that provides it is known as auxiliary variable. This additional/auxiliary information is utilized to estimate the unknown parameters at various times in the survey sampling process (Montgomery (2009a).

Haq (2013) suggested the New Hybrid-EWMA (HEWMA) chart for the process mean. The control chart's performance is monitored using the Average Run Length (ARL). The Average Run Length is the average number of points drawn on a control chart before an out of control condition is signaled. Montgomery (2009) considers the control chart with the lowest  $ARL_1$  score to be one of the best options for detecting assignable cause. Rayleigh distribution, one of the well-known continuous distributions

created by Lord Rayleigh, was discussed by Hussain and Ahmad (2014). Raza et al. (2015) suggested EWMA and DEWMA control charts for gamma-distributed lifetimes with Type-I censoring.

The concept of conditional anticipated values was employed to keep track of the mean level. The average run length was used as a metric in the performance assessments. Raza et al. (2016) wanted to look at EWMA control charts with Type-I censoring for Poisson-exponential lifetimes. Simple random sampling and rank set sampling, two extensively used sample strategies, were utilised. Monitoring mean level shifts with censored data piqued the interest of numerous applicable challenges.

Alkahtani (2013) investigated the effect of non-normality on the average run length (ARL) performance of EWMA and DEWMA using various skewed (Gamma) and symmetric non-normal (t) distributions. The paper dealt with the modelling of heterogeneity in lifetime processes using the mixture of the inverse Rayleigh distribution, and the focus was on the Bayesian inference of the mixture model using non-informative (Jeffreys and uniform) and informative (gamma) priors. Ali (2015), Abdelkhalek (2020), Alevizakos and Koukouvinos (2020) and Shahid and Rahee (2019) proposed and studied a new flexible extension of the inverse Rayleigh model.

Abbas et al. (2017) explained process monitoring is an essential element for an improved quality of final products. A variety of tools were used for it; control charts were one of these choices.

Raza et al. (2018), Abbasi et al. (2018) stated the process structures of manufacturing industry efficiently modeled control charts. Kowsalya et al. (2019) developed a new attribute control chart based on inverse Rayleigh distribution under a time truncated life test. Feng et al. (2020) proposed a test statistic based on the “divide-and-conquer” strategy, and integrated this into the multivariate EWMA charting scheme for Phase II process monitoring.

The main ideology was to calculate and combine the  $T_2$  statistics on low-dimensional sub-vectors together. Essentially the proposed procedure was distribution-free and computation efficient. Raza et al. (2020) proposed HDEWMA chart to monitor the mean of Weibull distribution in the presence of type-I censored data. Their focus was to use the conditional median (CM) for the imputation of censored observations. The performance of the control chart was determined by the average run length (ARL). They compared CM-DEWMA control chart with CM-based HDEWMA control chart.

In this paper we propose a Hybrid Double Exponentially Weighted Moving Average HDEWMA control chart by inverse Rayleigh distribution. The performance of the proposed chart will be determined using out-of-control-Average Run Length ( $ARL_1$ ).

## 2. METHODOLOGY

### 2.1 The Proposed Hybrid Double EWMA Control Chart

For monitoring the data generated from Inverse Rayleigh Distribution we define a hybrid DEWMA control chart. The PDF of the inverse Rayleigh distribution with scale parameter  $\theta$  is:

$$f(x) = \left(\frac{2\theta}{x^3}\right) \exp\left(-\frac{\theta}{x^2}\right) \quad x, \theta > 0 \tag{2.1.1}$$

The corresponding cumulative distribution function is,

$$F(x) = \exp\left(-\frac{\theta}{x^2}\right) \quad x, \theta > 0 \tag{2.1.2}$$

Let  $X_1, X_2, \dots$ , be a sequence of IID random variables from the Inverse Rayleigh Distribution. For Smoothing Parameters  $\lambda_1, \lambda_2, \lambda_3 \in [0, 1]$ , we define two new sequences  $E_1, E_2, \dots$ , and  $HE_1, HE_2, \dots$ , as follow:

$$E_t = \lambda_1 \bar{X}_t + (1 - \lambda_1) E_{t-1} \tag{2.1.3}$$

Now, the plotting statistic of DEWMA is calculated as:

$$HE_t = \lambda_2 E_t + (1 - \lambda_2) HE_{t-1} \tag{2.1.4}$$

Now, we define control charting statistics for Hybrid DEWMA, that is as follow:

$$HDE_t = \lambda_3 HE_t + (1 - \lambda_3) HDE_{t-1} \tag{2.1.5}$$

Compute the HDEWMA statistics for different subgroup size  $m$  and sample size  $n$ . Take the  $(1 - p)$ -th quantile point of the charting statistics values to define the UCL of the chart. Where,  $p$  is specified as 0.0027. (Montgomery et al. 1994).

The control limits of the proposed chart will be derived as:

$$E(HDE_t) = \mu$$

And variance of  $HE_t$ (cf. Haq, 2017).

$$V(HDE_t) = \frac{\lambda_1^2 \lambda_2^2 MSE}{(\lambda_1 - \lambda_2)^2} \left[ \frac{(1 - \lambda_1)^2(1 - (1 - \lambda_1)^{2i})}{1 - (1 - \lambda_1)^2} + \frac{(1 - \lambda_2)^2(1 - (1 - \lambda_2)^{2i})}{1 - (1 - \lambda_2)^2} \right] - \frac{2(1 - \lambda_1)(1 - \lambda_2)\{1 - (1 - \lambda_1)^i(1 - \lambda_2)^i\}}{1 - (1 - \lambda_1)(1 - \lambda_2)}$$

So control limits are:

$$UCL = \mu_0 + L_1 \sqrt{Var(HDE_t)}$$

$$LCL = \mu_0 - L_2 \sqrt{Var(HDE_t)}$$

Where  $L_1$  and  $L_2$  shows the 99.73 quantile point of the distribution under study.

**Algorithm 1:**

1. Initiate the algorithm by fixing the sample size,  $n$  and group size  $m$ . Estimate the parameter using phase-I data if it is unknown.
2. The sample is selected using SRS and RSS schemes.
3. Calculate the charting statistics of proposed HDEWMA statistics for given sample size “ $n$ ”. Now take  $(1 - p)$ th quantile point, here  $\alpha$  is predefined,
4. Repeat Step No.2 1000 number of time & compute the average, it will out-turn the Upper Control Limit.
5. Hatch the values of HDEWMA monitoring statistic with respect to the subgroup numbers. Record the sample number at which the monitoring statistic falls outside the control limit,
6. Repeat Step 4 a large number of times and calculate the mean of the subgroup numbers at which the process first time crossed the UCL. This will result in  $ARL_0$  if the subgroups are generated from the in-control process. For the out-of-control ARL computation, introduce a shift in the data and test it against the control limit constructed using in-control data. Repeat steps 4-5 and calculate the average of subgroups falling outside the UCL, which is  $ARL_1$ .

**3. RESULTS AND DISCUSSION**

To study the features of the proposed Hybrid DEWMA control chart, we have simulated the data and applied the Hybrid DEWMA chart. The procedure proposed in section 2 is evaluated using SRS and RSS. The EWMA weight ( $\lambda_1$ ) and HEWMA weight ( $\lambda_2$ ) are fixed for evaluation purpose. The simulated distribution is found to follow Inverse Rayleigh distribution. Therefore, the limits are based on Inverse Rayleigh distribution. Among these, some results are presented here for discussion purposes. Table 3.1 gives the ARLs of the proposed HDEWMA control chart.

**Table 3.1 ARL<sub>1</sub> for HDEWMA using SRS and RSS with  $\theta=1.5$ ,  $ARL_0=100$ .**

| Shift |         | HDEWMA (SRS) |         | HDEWMA (RSS) |
|-------|---------|--------------|---------|--------------|
| 10%   | Average | 13.476       | Average | 10.601       |
|       | SD      | 23.3350      | SD      | 22.5923      |
| 15%   | Average | 12.455       | Average | 10.140       |
|       | SD      | 25.3036      | SD      | 24.4515      |
| 20%   | Average | 11.719       | Average | 9.791        |
|       | SD      | 25.0220      | SD      | 22.7265      |

|     |         |         |         |         |
|-----|---------|---------|---------|---------|
| 30% | Average | 10.456  | Average | 9.044   |
|     | SD      | 25.3174 | SD      | 23.2727 |
| 40% | Average | 9.348   | Average | 8.444   |
|     | SD      | 26.1337 | SD      | 22.8477 |

Table 3.1 represents the out of control values of ARL for HDEWMA (SRS) & HDEWMA (RSS) control charts of sample size  $n=3$ . The samples are generated from Inverse Rayleigh Distribution with parameter  $\theta=1.5$ . The in-control ARL ( $ARL_0$ ) is fixed at 100, We have simulated results 10,000 times using Monte Carlo Simulations (MC Simulation).

It is noticed that with a 10% increase in shift, the  $ARL_1$  value is found to be 13.476 with SDRL is 23.335 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the  $ARL_1$  is 10.601 with SDRL is 22.5923. With 15% increase in shift, the  $ARL_1$  value is found to be 12.455 with SDRL=25.3036 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the  $ARL_1$  value is 10.140 with SDRL = 24.4515. With 20% increase in shift, the  $ARL_1$  value is found to be 11.719 with SDRL= 25.0220 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the  $ARL_1$  is 9.791 with SDRL = 22.7265. With 30% increase in shift, the  $ARL_1$  value is found to be 10.456 with SDRL = 25.3174 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the  $ARL_1$  is 9.044 with SDRL = 23.2727. With 40% increase in shift, the  $ARL_1$  value is found to be 9.348 with SDRL=26.1337 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the  $ARL_1$  is 8.444 with SDRL = 22.8477. We can see that, for increase in shift, the  $ARL_1$  values for HDEWMA (RSS) control chart is less than HDEWMA (SRS) control chart, which shows HDEWMA using RSS performs better than the HDEWMA control chart using SRS.

**Figure 3.1**  $ARL_1$  for HDEWMA using SRS and RSS with  $\theta=1.5$ ,  $ARL_0=100$ .

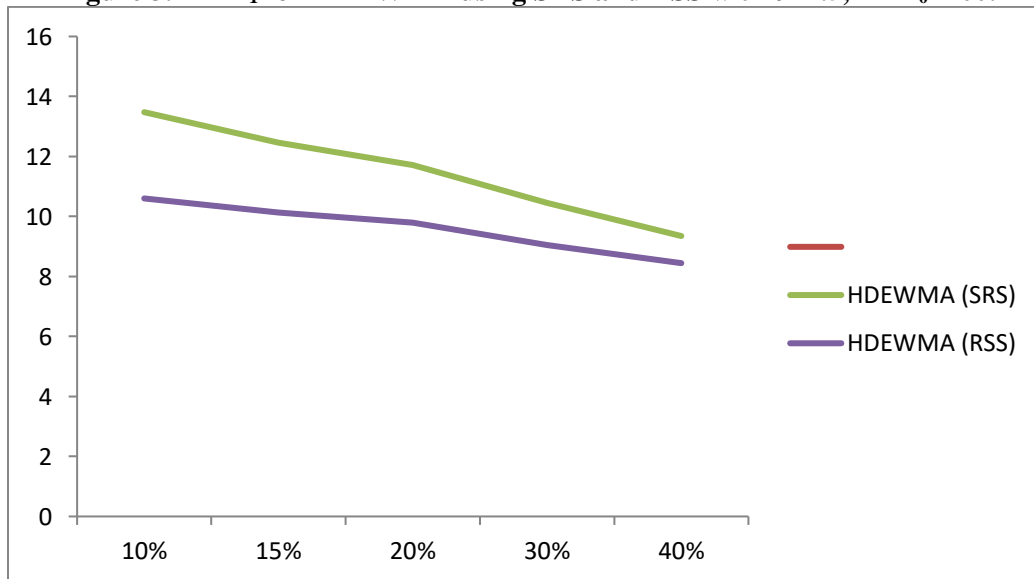


Figure 3.1 graphically represents the out-of-control values of ARL for HDEWMA (SRS) & HDEWMA (RSS) control charts of sample size  $n=3$ . The samples are generated from Inverse Reyleigh Distribution with parameter  $\theta=1.5$ . The in control ARL ( $ARL_0$ ) is fixed at 100, We have simulated results 10,000 times using MC Simulations.

It is noticed that with 10% increase, the  $ARL_1$  value is found to be 13.476 with SDRL is 23.335 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the  $ARL_1$  is 10.601 with SDRL is 22.5923. With 15% increase in shift, the  $ARL_1$  value is found to be 12.455 with SDRL=25.3036 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the  $ARL_1$  value is 10.140 with SDRL = 24.4515. With 20% increase in shift, the  $ARL_1$  value is found to be 11.719 with SDRL= 25.0220 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the  $ARL_1$  is 9.791 with SDRL = 22.7265. With 30% increase in shift, the  $ARL_1$  value is found to be 10.456 with SDRL = 25.3174 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the  $ARL_1$  is 9.044 with SDRL = 23.2727. With 40% increase in shift, the  $ARL_1$  value is found to be 9.348 with SDRL=26.1337 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the  $ARL_1$

is 8.444 with SDRL = 22.8477. We can see that, for increase in shift, the  $ARL_1$  values for HDEWMA (RSS) control chart is less than HDEWMA (SRS) control chart, which shows The performance of the HDEWMA (RSS) is better than the HDEWMA (SRS).

**Table 3.1.a Comparison of  $ARL_1$  for HDEWMA, EWMA & CUSUM using SRS and RSS when  $\theta=1.5, ARL_0=100$ .**

| Shift | HDEWMA (SRS) | HDEWMA (RSS) | EWMA (SRS) | EWMA (RSS) | CUSUM (SRS) | CUSUM (RSS) |
|-------|--------------|--------------|------------|------------|-------------|-------------|
| 10%   | Avg          | 13.47        | 10.60      | 15.53      | 13.71       | 14.62       |
|       | SD           | 23.33        | 22.59      | 25.08      | 25.11       | 24.09       |
| 15%   | Avg          | 12.45        | 10.14      | 14.25      | 13.21       | 13.87       |
|       | SD           | 25.30        | 24.45      | 24.33      | 24.08       | 23.15       |
| 20%   | Avg          | 11.71        | 9.79       | 13.61      | 12.67       | 12.89       |
|       | SD           | 25.02        | 22.72      | 23.31      | 24.01       | 22.79       |
| 30%   | Avg          | 10.45        | 9.04       | 12.50      | 10.81       | 11.66       |
|       | SD           | 25.31        | 23.27      | 22.22      | 22.76       | 22.26       |
| 40%   | Avg          | 9.34         | 8.44       | 11.29      | 9.70        | 10.48       |
|       | SD           | 26.13        | 22.84      | 22.09      | 22.13       | 21.05       |

Table 3.1.a compares the out of control values of ARL for HDEWMA (SRS) and HDEWMA (RSS) with EWMA and CUSUM control charts for sample size  $n=3$ . The samples are generated from Inverse Rayleigh Distribution with parameter  $\theta=1.5$ . The in control ARL ( $ARL_0$ ) is fixed at 100, We have simulated results 10,000 times using MC Simulations.

We can see that, when the shift is increased, the  $ARL_1$  values for HDEWMA control charts using rank set sampling is less than simple random sampling, which shows HDEWMA (RSS) control chart performs better than the HDEWMA (SRS) control chart. Also, it can be observed that the HDEWMA (RSS) shows minimum ARL values among the other competitor control charts i.e EWMA & CUSUM. So we can say that HDEWMA (RSS) is the most efficient control chart for the detection of shift among these. It shows better performance than EWMA and CUSUM control charts.

**Table 3.2 Simulated Samples and Calculation of statistics of HDEWMA chart**

| Inverse Rayleigh R.No |       |       | HDE WMA Statistis | Inverse Rayleigh R.No |       |       | HDE WMA Statistis | Inverse Rayleigh R.No |       |       | HDE WMA Statistis |
|-----------------------|-------|-------|-------------------|-----------------------|-------|-------|-------------------|-----------------------|-------|-------|-------------------|
| 2.382                 | 0.682 | 3.363 | 1.8337            | 3.215                 | 2.971 | 0.493 | 1.2333            | 3.158                 | 1.882 | 1.656 | 1.2898            |
| 953                   | 294   | 658   | 06                | 87                    | 659   | 318   | 89                | 38                    | 056   | 595   | 57                |
| 2.051                 | 2.089 | 2.419 | 1.2130            | 0.714                 | 3.008 | 2.402 | 1.4930            | 2.520                 | 3.238 | 2.363 | 2.2606            |
| 321                   | 189   | 881   | 11                | 164                   | 044   | 361   | 23                | 815                   | 732   | 489   | 69                |
| 2.618                 | 1.871 | 0.821 | 1.2130            | 1.734                 | 0.794 | 4.485 | 0.2131            | 2.967                 | 1.897 | 0.802 | 1.9132            |
| 439                   | 851   | 133   | 11                | 271                   | 984   | 471   | 4                 | 042                   | 054   | 98    | 2                 |
| 1.480                 | 0.433 | 3.902 | 1.8071            | 2.335                 | 2.858 | 2.213 | 1.4925            | 2.298                 | 0.468 | 2.120 | 2.1895            |
| 128                   | 819   | 656   | 91                | 75                    | 507   | 81    | 34                | 446                   | 165   | 194   | 82                |
| 1.578                 | 2.433 | 2.793 | 2.4251            | 3.466                 | 0.503 | 1.505 | 1.9532            | 1.459                 | 0.880 | 2.936 |                   |
| 465                   | 43    | 356   | 66                | 55                    | 696   | 475   | 34                | 319                   | 196   | 515   | 5.107             |
| 0.548                 | 1.890 | 2.625 |                   | 2.617                 | 1.656 | 1.590 |                   | 1.078                 | 2.190 | 3.635 | 2.0705            |
| 548                   | 562   | 547   | 5.107             | 836                   | 037   | 839   | 5.107             | 483                   | 758   | 592   | 34                |
| 0.629                 | 0.805 | 1.995 | 1.0261            | 2.516                 | 1.958 | 3.559 | 2.1417            | 2.295                 | 3.768 | 1.977 | 1.7218            |
| 757                   | 34    | 672   | 57                | 016                   | 422   | 974   | 79                | 042                   | 096   | 212   | 05                |
| 1.245                 | 1.718 | 1.833 |                   | 2.865                 | 1.105 | 1.233 |                   | 1.693                 | 1.769 | 1.289 | 1.6011            |
| 913                   | 476   | 706   | 5.107             | 441                   | 934   | 389   | 5.107             | 961                   | 089   | 857   | 36                |
| 0.412                 | 0.790 | 1.213 | 0.2556            | 1.357                 | 4.034 | 1.493 | 2.2595            | 2.579                 | 2.814 | 2.260 | 1.5217            |
| 014                   | 938   | 011   | 81                | 295                   | 043   | 023   | 83                | 819                   | 422   | 669   | 77                |
| 0.726                 | 0.177 | 2.295 | 1.6565            | 1.556                 | 1.219 | 0.213 | 1.8762            | 1.659                 | 2.681 | 1.913 |                   |
| 176                   | 768   | 66    | 95                | 248                   | 095   | 14    | 04                | 297                   | 053   | 22    | 5.107             |

|       |       |       |        |       |       |       |        |       |       |       |        |
|-------|-------|-------|--------|-------|-------|-------|--------|-------|-------|-------|--------|
| 1.997 | 2.187 | 1.807 | 2.3634 | 2.128 | 1.724 | 1.492 |        | 2.382 | 1.763 | 2.189 | 1.1211 |
| 629   | 867   | 191   | 89     | 116   | 023   | 534   | 5.107  | 953   | 74    | 582   | 47     |
| 1.191 | 1.924 | 2.425 | 0.8029 | 1.286 | 2.961 | 1.953 | 1.4243 | 2.783 | 0.681 | 3.528 | 0.4933 |
| 937   | 113   | 166   | 8      | 818   | 882   | 234   | 62     | 719   | 86    | 656   | 18     |
| 3.692 | 1.586 | 4.803 | 2.1201 | 4.314 | 1.466 | 3.143 | 1.3736 | 3.702 | 4.652 | 2.070 | 2.4023 |
| 983   | 588   | 78    | 94     | 494   | 767   | 783   | 74     | 232   | 499   | 534   | 61     |
| 1.684 | 2.117 | 1.026 |        | 0.364 | 1.915 | 2.141 | 2.1618 | 2.398 | 1.564 | 1.721 |        |
| 446   | 991   | 157   | 5.107  | 438   | 257   | 779   | 03     | 368   | 775   | 805   | 5.107  |
| 1.803 | 3.191 | 2.654 |        | 2.961 | 0.704 | 2.696 | 1.2322 | 2.209 | 1.556 | 1.601 | 2.2138 |
| 541   | 923   | 436   | 5.107  | 845   | 21    | 256   | 47     | 012   | 516   | 136   | 1      |
| 2.217 | 1.364 | 0.255 | 1.9772 | 2.450 | 1.823 | 2.259 |        | 2.865 | 1.105 | 1.233 |        |
| 185   | 228   | 681   | 12     | 837   | 29    | 583   | 5.107  | 441   | 934   | 389   | 5.107  |
| 3.158 | 1.882 | 1.656 | 1.2898 | 1.819 | 1.257 | 1.876 | 1.4766 | 1.357 | 4.034 | 1.493 | 2.2595 |
| 38    | 056   | 595   | 57     | 483   | 233   | 204   | 91     | 295   | 043   | 023   | 83     |
| 2.520 | 3.238 | 2.363 | 2.2606 | 3.532 | 1.695 | 3.860 | 1.5009 | 1.556 | 1.219 | 0.213 | 1.8762 |
| 815   | 732   | 489   | 69     | 945   | 871   | 049   | 75     | 248   | 095   | 14    | 04     |
| 2.967 | 1.897 | 0.802 | 1.9132 | 2.136 | 4.096 | 1.424 | 1.4404 | 2.128 | 1.724 | 1.492 |        |
| 042   | 054   | 98    | 2      | 282   | 41    | 362   | 88     | 116   | 023   | 534   | 5.107  |
| 2.298 | 0.468 | 2.120 | 2.1895 | 1.591 | 3.597 | 1.373 | 1.2710 | 1.286 | 2.961 | 1.953 | 1.4243 |
| 446   | 165   | 194   | 82     | 475   | 052   | 674   | 59     | 818   | 882   | 234   | 62     |
| 1.459 | 0.880 | 2.936 |        | 1.341 | 2.994 | 2.161 | 0.8353 | 4.314 | 1.466 | 3.143 | 1.3736 |
| 319   | 196   | 515   | 5.107  | 987   | 698   | 803   | 42     | 494   | 767   | 783   | 74     |
| 1.078 | 2.190 | 3.635 | 2.0705 | 3.460 | 2.332 | 1.232 |        | 0.364 | 1.915 | 2.141 | 2.1618 |
| 483   | 758   | 592   | 34     | 82    | 72    | 247   | 5.107  | 438   | 257   | 779   | 03     |
| 2.295 | 3.768 | 1.977 | 1.7218 | 2.645 | 1.980 | 3.755 | 1.9465 | 2.961 | 0.704 | 2.696 | 1.2322 |
| 042   | 096   | 212   | 05     | 253   | 547   | 891   | 21     | 845   | 21    | 256   | 47     |
| 1.693 | 1.769 | 1.289 | 1.6011 | 2.057 | 3.711 | 1.476 |        | 2.450 | 1.823 | 2.259 |        |
| 961   | 089   | 857   | 36     | 372   | 336   | 691   | 5.107  | 837   | 29    | 583   | 5.107  |
| 2.579 | 2.814 | 2.260 | 1.5217 | 0.924 | 2.569 | 1.500 | 1.5126 | 1.819 | 1.257 | 1.876 | 1.4766 |
| 819   | 422   | 669   | 77     | 725   | 745   | 975   | 82     | 483   | 233   | 204   | 91     |
| 1.659 | 2.681 | 1.913 |        | 1.474 | 2.583 | 1.440 | 0.6735 | 3.532 | 1.695 | 3.860 | 1.5009 |
| 297   | 053   | 22    | 5.107  | 531   | 619   | 488   | 83     | 945   | 871   | 049   | 75     |
| 2.382 | 1.763 | 2.189 | 1.1211 | 0.634 | 1.298 | 1.271 |        | 2.136 | 4.096 | 1.424 | 1.4404 |
| 953   | 74    | 582   | 47     | 185   | 928   | 059   | 5.107  | 282   | 41    | 362   | 88     |
| 2.783 | 0.681 | 3.528 | 0.4933 | 1.054 | 2.279 | 0.835 | 1.1797 | 1.591 | 3.597 | 1.373 | 1.2710 |
| 719   | 86    | 656   | 18     | 286   | 96    | 342   | 96     | 475   | 052   | 674   | 59     |
| 3.702 | 4.652 | 2.070 | 2.4023 | 1.674 | 4.397 | 2.720 | 1.3145 | 1.341 | 2.994 | 2.161 | 0.8353 |
| 232   | 499   | 534   | 61     | 931   | 629   | 744   | 67     | 987   | 698   | 803   | 42     |
| 2.398 | 1.564 | 1.721 |        | 4.087 | 1.229 | 1.946 | 1.5794 | 3.460 | 2.332 | 1.232 |        |
| 368   | 775   | 805   | 5.107  | 779   | 768   | 521   | 2      | 82    | 72    | 247   | 5.107  |
| 2.209 | 1.556 | 1.601 | 2.2138 | 2.274 | 1.246 | 2.698 | 1.5095 | 2.645 | 1.980 | 3.755 | 1.9465 |
| 012   | 516   | 136   | 1      | 22    | 049   | 305   | 25     | 253   | 547   | 891   | 21     |
| 0.412 | 4.916 | 1.521 | 1.5054 | 3.236 | 3.220 | 1.512 | 0.9794 | 2.057 | 3.711 | 1.476 |        |
| 404   | 563   | 777   | 75     | 14    | 256   | 682   | 32     | 372   | 336   | 691   | 5.107  |
| 0.412 | 0.790 | 1.213 | 0.2556 | 1.357 | 4.034 | 1.493 | 2.2595 | 2.579 | 2.814 | 2.260 | 1.5217 |
| 014   | 938   | 011   | 81     | 295   | 043   | 023   | 83     | 819   | 422   | 669   | 77     |
| 0.726 | 0.177 | 2.295 | 1.6565 | 1.556 | 1.219 | 0.213 | 1.8762 | 1.659 | 2.681 | 1.913 |        |
| 176   | 768   | 66    | 95     | 248   | 095   | 14    | 04     | 297   | 053   | 22    | 5.107  |
| 1.997 | 2.187 | 1.807 | 2.3634 | 2.128 | 1.724 | 1.492 |        | 2.382 | 1.763 | 2.189 | 1.1211 |
| 629   | 867   | 191   | 89     | 116   | 023   | 534   | 5.107  | 953   | 74    | 582   | 47     |
| 1.191 | 1.924 | 2.425 | 0.8029 | 1.286 | 2.961 | 1.953 | 1.4243 | 2.783 | 0.681 | 3.528 | 0.4933 |
| 937   | 113   | 166   | 8      | 818   | 882   | 234   | 62     | 719   | 86    | 656   | 18     |

Hybrid Double Exponentially Weighted Moving Average (HDEWMA) Control Chart for Inverse Rayleigh Distribution

|       |       |       |        |       |       |       |        |       |       |       |        |
|-------|-------|-------|--------|-------|-------|-------|--------|-------|-------|-------|--------|
| 3.692 | 1.586 | 4.803 | 2.1201 | 4.314 | 1.466 | 3.143 | 1.3736 | 3.702 | 4.652 | 2.070 | 2.4023 |
| 983   | 588   | 78    | 94     | 494   | 767   | 783   | 74     | 232   | 499   | 534   | 61     |
| 1.684 | 2.117 | 1.026 |        | 0.364 | 1.915 | 2.141 | 2.1618 | 2.398 | 1.564 | 1.721 |        |
| 446   | 991   | 157   | 5.107  | 438   | 257   | 779   | 03     | 368   | 775   | 805   | 5.107  |
| 1.803 | 3.191 | 2.654 |        | 2.961 | 0.704 | 2.696 | 1.2322 | 2.209 | 1.556 | 1.601 | 2.2138 |
| 541   | 923   | 436   | 5.107  | 845   | 21    | 256   | 47     | 012   | 516   | 136   | 1      |
| 2.217 | 1.364 | 0.255 | 1.9772 | 2.450 | 1.823 | 2.259 |        | 2.865 | 1.105 | 1.233 |        |
| 185   | 228   | 681   | 12     | 837   | 29    | 583   | 5.107  | 441   | 934   | 389   | 5.107  |
| 3.158 | 1.882 | 1.656 | 1.2898 | 1.819 | 1.257 | 1.876 | 1.4766 | 1.357 | 4.034 | 1.493 | 2.2595 |
| 38    | 056   | 595   | 57     | 483   | 233   | 204   | 91     | 295   | 043   | 023   | 83     |
| 2.520 | 3.238 | 2.363 | 2.2606 | 3.532 | 1.695 | 3.860 | 1.5009 | 1.556 | 1.219 | 0.213 | 1.8762 |
| 815   | 732   | 489   | 69     | 945   | 871   | 049   | 75     | 248   | 095   | 14    | 04     |
| 2.967 | 1.897 | 0.802 | 1.9132 | 2.136 | 4.096 | 1.424 | 1.4404 | 2.128 | 1.724 | 1.492 |        |
| 042   | 054   | 98    | 2      | 282   | 41    | 362   | 88     | 116   | 023   | 534   | 5.107  |
| 2.298 | 0.468 | 2.120 | 2.1895 | 1.591 | 3.597 | 1.373 | 1.2710 | 1.286 | 2.961 | 1.953 | 1.4243 |
| 446   | 165   | 194   | 82     | 475   | 052   | 674   | 59     | 818   | 882   | 234   | 62     |
| 1.459 | 0.880 | 2.936 |        | 1.341 | 2.994 | 2.161 | 0.8353 | 4.314 | 1.466 | 3.143 | 1.3736 |
| 319   | 196   | 515   | 5.107  | 987   | 698   | 803   | 42     | 494   | 767   | 783   | 74     |
| 1.078 | 2.190 | 3.635 | 2.0705 | 3.460 | 2.332 | 1.232 |        | 0.364 | 1.915 | 2.141 | 2.1618 |
| 483   | 758   | 592   | 34     | 82    | 72    | 247   | 5.107  | 438   | 257   | 779   | 03     |
| 2.295 | 3.768 | 1.977 | 1.7218 | 2.645 | 1.980 | 3.755 | 1.9465 | 2.961 | 0.704 | 2.696 | 1.2322 |
| 042   | 096   | 212   | 05     | 253   | 547   | 891   | 21     | 845   | 21    | 256   | 47     |
| 1.693 | 1.769 | 1.289 | 1.6011 | 2.057 | 3.711 | 1.476 |        | 2.450 | 1.823 | 2.259 |        |
| 961   | 089   | 857   | 36     | 372   | 336   | 691   | 5.107  | 837   | 29    | 583   | 5.107  |
| 2.579 | 2.814 | 2.260 | 1.5217 | 0.924 | 2.569 | 1.500 | 1.5126 | 1.819 | 1.257 | 1.876 | 1.4766 |
| 819   | 422   | 669   | 77     | 725   | 745   | 975   | 82     | 483   | 233   | 204   | 91     |
| 1.659 | 2.681 | 1.913 |        | 1.474 | 2.583 | 1.440 | 0.6735 | 3.532 | 1.695 | 3.860 | 1.5009 |
| 297   | 053   | 22    | 5.107  | 531   | 619   | 488   | 83     | 945   | 871   | 049   | 75     |
| 2.382 | 1.763 | 2.189 | 1.1211 | 0.634 | 1.298 | 1.271 |        | 2.136 | 4.096 | 1.424 | 1.4404 |
| 953   | 74    | 582   | 47     | 185   | 928   | 059   | 5.107  | 282   | 41    | 362   | 88     |
| 2.783 | 0.681 | 3.528 | 0.4933 | 1.054 | 2.279 | 0.835 | 1.1797 | 1.591 | 3.597 | 1.373 | 1.2710 |
| 719   | 86    | 656   | 18     | 286   | 96    | 342   | 96     | 475   | 052   | 674   | 59     |
| 3.702 | 4.652 | 2.070 | 2.4023 | 1.674 | 4.397 | 2.720 | 1.3145 | 1.341 | 2.994 | 2.161 | 0.8353 |
| 232   | 499   | 534   | 61     | 931   | 629   | 744   | 67     | 987   | 698   | 803   | 42     |
| 2.398 | 1.564 | 1.721 |        | 4.087 | 1.229 | 1.946 | 1.5794 | 3.460 | 2.332 | 1.232 |        |
| 368   | 775   | 805   | 5.107  | 779   | 768   | 521   | 2      | 82    | 72    | 247   | 5.107  |
| 2.209 | 1.556 | 1.601 | 2.2138 | 2.274 | 1.246 | 2.698 | 1.5095 | 2.645 | 1.980 | 3.755 | 1.9465 |
| 012   | 516   | 136   | 1      | 22    | 049   | 305   | 25     | 253   | 547   | 891   | 21     |
| 0.412 | 4.916 | 1.521 | 1.5054 | 3.236 | 3.220 | 1.512 | 0.9794 | 2.057 | 3.711 | 1.476 |        |
| 404   | 563   | 777   | 75     | 14    | 256   | 682   | 32     | 372   | 336   | 691   | 5.107  |
| 1.097 | 1.222 | 5.066 | 1.5908 | 2.154 | 3.718 | 0.673 | 2.1745 | 0.924 | 2.569 | 1.500 | 1.5126 |
| 086   | 473   | 057   | 39     | 742   | 465   | 583   | 55     | 725   | 745   | 975   | 82     |
| 3.158 | 1.882 | 1.656 | 1.2898 | 1.819 | 1.257 | 1.876 | 1.4766 | 1.357 | 4.034 | 1.493 | 2.2595 |
| 38    | 056   | 595   | 57     | 483   | 233   | 204   | 91     | 295   | 043   | 023   | 83     |
| 2.520 | 3.238 | 2.363 | 2.2606 | 3.532 | 1.695 | 3.860 | 1.5009 | 1.556 | 1.219 | 0.213 | 1.8762 |
| 815   | 732   | 489   | 69     | 945   | 871   | 049   | 75     | 248   | 095   | 14    | 04     |
| 2.967 | 1.897 | 0.802 | 1.9132 | 2.136 | 4.096 | 1.424 | 1.4404 | 2.128 | 1.724 | 1.492 |        |
| 042   | 054   | 98    | 2      | 282   | 41    | 362   | 88     | 116   | 023   | 534   | 5.107  |
| 2.298 | 0.468 | 2.120 | 2.1895 | 1.591 | 3.597 | 1.373 | 1.2710 | 1.286 | 2.961 | 1.953 | 1.4243 |
| 446   | 165   | 194   | 82     | 475   | 052   | 674   | 59     | 818   | 882   | 234   | 62     |
| 1.459 | 0.880 | 2.936 |        | 1.341 | 2.994 | 2.161 | 0.8353 | 4.314 | 1.466 | 3.143 | 1.3736 |
| 319   | 196   | 515   | 5.107  | 987   | 698   | 803   | 42     | 494   | 767   | 783   | 74     |

|       |       |       |        |       |       |       |        |       |       |       |        |
|-------|-------|-------|--------|-------|-------|-------|--------|-------|-------|-------|--------|
| 1.078 | 2.190 | 3.635 | 2.0705 | 3.460 | 2.332 | 1.232 |        | 0.364 | 1.915 | 2.141 | 2.1618 |
| 483   | 758   | 592   | 34     | 82    | 72    | 247   | 5.107  | 438   | 257   | 779   | 03     |
| 2.295 | 3.768 | 1.977 | 1.7218 | 2.645 | 1.980 | 3.755 | 1.9465 | 2.961 | 0.704 | 2.696 | 1.2322 |
| 042   | 096   | 212   | 05     | 253   | 547   | 891   | 21     | 845   | 21    | 256   | 47     |
| 1.693 | 1.769 | 1.289 | 1.6011 | 2.057 | 3.711 | 1.476 |        | 2.450 | 1.823 | 2.259 |        |
| 961   | 089   | 857   | 36     | 372   | 336   | 691   | 5.107  | 837   | 29    | 583   | 5.107  |
| 2.579 | 2.814 | 2.260 | 1.5217 | 0.924 | 2.569 | 1.500 | 1.5126 | 1.819 | 1.257 | 1.876 | 1.4766 |
| 819   | 422   | 669   | 77     | 725   | 745   | 975   | 82     | 483   | 233   | 204   | 91     |
| 1.659 | 2.681 | 1.913 |        | 1.474 | 2.583 | 1.440 | 0.6735 | 3.532 | 1.695 | 3.860 | 1.5009 |
| 297   | 053   | 22    | 5.107  | 531   | 619   | 488   | 83     | 945   | 871   | 049   | 75     |
| 2.382 | 1.763 | 2.189 | 1.1211 | 0.634 | 1.298 | 1.271 |        | 2.136 | 4.096 | 1.424 | 1.4404 |
| 953   | 74    | 582   | 47     | 185   | 928   | 059   | 5.107  | 282   | 41    | 362   | 88     |
| 2.783 | 0.681 | 3.528 | 0.4933 | 1.054 | 2.279 | 0.835 | 1.1797 | 1.591 | 3.597 | 1.373 | 1.2710 |
| 719   | 86    | 656   | 18     | 286   | 96    | 342   | 96     | 475   | 052   | 674   | 59     |
| 3.702 | 4.652 | 2.070 | 2.4023 | 1.674 | 4.397 | 2.720 | 1.3145 | 1.341 | 2.994 | 2.161 | 0.8353 |
| 232   | 499   | 534   | 61     | 931   | 629   | 744   | 67     | 987   | 698   | 803   | 42     |
| 2.398 | 1.564 | 1.721 |        | 4.087 | 1.229 | 1.946 | 1.5794 | 3.460 | 2.332 | 1.232 |        |
| 368   | 775   | 805   | 5.107  | 779   | 768   | 521   | 2      | 82    | 72    | 247   | 5.107  |
| 2.209 | 1.556 | 1.601 | 2.2138 | 2.274 | 1.246 | 2.698 | 1.5095 | 2.645 | 1.980 | 3.755 | 1.9465 |
| 012   | 516   | 136   | 1      | 22    | 049   | 305   | 25     | 253   | 547   | 891   | 21     |
| 0.412 | 4.916 | 1.521 | 1.5054 | 3.236 | 3.220 | 1.512 | 0.9794 | 2.057 | 3.711 | 1.476 |        |
| 404   | 563   | 777   | 75     | 14    | 256   | 682   | 32     | 372   | 336   | 691   | 5.107  |
| 0.412 | 0.790 | 1.213 | 0.2556 | 1.357 | 4.034 | 1.493 | 2.2595 | 2.579 | 2.814 | 2.260 | 1.5217 |
| 014   | 938   | 011   | 81     | 295   | 043   | 023   | 83     | 819   | 422   | 669   | 77     |
| 0.726 | 0.177 | 2.295 | 1.6565 | 1.556 | 1.219 | 0.213 | 1.8762 | 1.659 | 2.681 | 1.913 |        |
| 176   | 768   | 66    | 95     | 248   | 095   | 14    | 04     | 297   | 053   | 22    | 5.107  |
| 1.997 | 2.187 | 1.807 | 2.3634 | 2.128 | 1.724 | 1.492 |        | 2.382 | 1.763 | 2.189 | 1.1211 |
| 629   | 867   | 191   | 89     | 116   | 023   | 534   | 5.107  | 953   | 74    | 582   | 47     |
| 1.191 | 1.924 | 2.425 | 0.8029 | 1.286 | 2.961 | 1.953 | 1.4243 | 2.783 | 0.681 | 3.528 | 0.4933 |
| 937   | 113   | 166   | 8      | 818   | 882   | 234   | 62     | 719   | 86    | 656   | 18     |
| 3.692 | 1.586 | 4.803 | 2.1201 | 4.314 | 1.466 | 3.143 | 1.3736 | 3.702 | 4.652 | 2.070 | 2.4023 |
| 983   | 588   | 78    | 94     | 494   | 767   | 783   | 74     | 232   | 499   | 534   | 61     |
| 1.684 | 2.117 | 1.026 |        | 0.364 | 1.915 | 2.141 | 2.1618 | 2.398 | 1.564 | 1.721 |        |
| 446   | 991   | 157   | 5.107  | 438   | 257   | 779   | 03     | 368   | 775   | 805   | 5.107  |
| 1.803 | 3.191 | 2.654 |        | 2.961 | 0.704 | 2.696 | 1.2322 | 2.209 | 1.556 | 1.601 | 2.2138 |
| 541   | 923   | 436   | 5.107  | 845   | 21    | 256   | 47     | 012   | 516   | 136   | 1      |
| 2.217 | 1.364 | 0.255 | 1.9772 | 2.450 | 1.823 | 2.259 |        | 2.865 | 1.105 | 1.233 |        |
| 185   | 228   | 681   | 12     | 837   | 29    | 583   | 5.107  | 441   | 934   | 389   | 5.107  |
| 3.158 | 1.882 | 1.656 | 1.2898 | 1.819 | 1.257 | 1.876 | 1.4766 | 1.357 | 4.034 | 1.493 | 2.2595 |
| 38    | 056   | 595   | 57     | 483   | 233   | 204   | 91     | 295   | 043   | 023   | 83     |
| 2.520 | 3.238 | 2.363 | 2.2606 | 3.532 | 1.695 | 3.860 | 1.5009 | 1.556 | 1.219 | 0.213 | 1.8762 |
| 815   | 732   | 489   | 69     | 945   | 871   | 049   | 75     | 248   | 095   | 14    | 04     |
| 2.579 | 2.814 | 2.260 | 1.5217 | 0.924 | 2.569 | 1.500 | 1.5126 | 1.819 | 1.257 | 1.876 | 1.4766 |
| 819   | 422   | 669   | 77     | 725   | 745   | 975   | 82     | 483   | 233   | 204   | 91     |
| 1.659 | 2.681 | 1.913 |        | 1.474 | 2.583 | 1.440 | 0.6735 | 3.532 | 1.695 | 3.860 | 1.5009 |
| 297   | 053   | 22    | 5.107  | 531   | 619   | 488   | 83     | 945   | 871   | 049   | 75     |
| 2.382 | 1.763 | 2.189 | 1.1211 | 0.634 | 1.298 | 1.271 |        | 2.136 | 4.096 | 1.424 | 1.4404 |
| 953   | 74    | 582   | 47     | 185   | 928   | 059   | 5.107  | 282   | 41    | 362   | 88     |
| 2.783 | 0.681 | 3.528 | 0.4933 | 1.054 | 2.279 | 0.835 | 1.1797 | 1.591 | 3.597 | 1.373 | 1.2710 |
| 719   | 86    | 656   | 18     | 286   | 96    | 342   | 96     | 475   | 052   | 674   | 59     |
| 3.702 | 4.652 | 2.070 | 2.4023 | 1.674 | 4.397 | 2.720 | 1.3145 | 1.341 | 2.994 | 2.161 | 0.8353 |
| 232   | 499   | 534   | 61     | 931   | 629   | 744   | 67     | 987   | 698   | 803   | 42     |



|       |       |       |        |       |       |       |        |       |       |       |        |
|-------|-------|-------|--------|-------|-------|-------|--------|-------|-------|-------|--------|
| 2.398 | 1.564 | 1.721 |        | 4.087 | 1.229 | 1.946 | 1.5794 | 3.460 | 2.332 | 1.232 |        |
| 368   | 775   | 805   | 5.107  | 779   | 768   | 521   | 2      | 82    | 72    | 247   | 5.107  |
| 2.209 | 1.556 | 1.601 | 2.2138 | 2.274 | 1.246 | 2.698 | 1.5095 | 2.645 | 1.980 | 3.755 | 1.9465 |
| 012   | 516   | 136   | 1      | 22    | 049   | 305   | 25     | 253   | 547   | 891   | 21     |
| 0.412 | 4.916 | 1.521 | 1.5054 | 3.236 | 3.220 | 1.512 | 0.9794 | 2.057 | 3.711 | 1.476 |        |
| 404   | 563   | 777   | 75     | 14    | 256   | 682   | 32     | 372   | 336   | 691   | 5.107  |
| 0.412 | 0.790 | 1.213 | 0.2556 | 1.357 | 4.034 | 1.493 | 2.2595 | 2.579 | 2.814 | 2.260 | 1.5217 |
| 014   | 938   | 011   | 81     | 295   | 043   | 023   | 83     | 819   | 422   | 669   | 77     |
| 0.726 | 0.177 | 2.295 | 1.6565 | 1.556 | 1.219 | 0.213 | 1.8762 | 1.659 | 2.681 | 1.913 |        |
| 176   | 768   | 66    | 95     | 248   | 095   | 14    | 04     | 297   | 053   | 22    | 5.107  |
| 1.997 | 2.187 | 1.807 | 2.3634 | 2.128 | 1.724 | 1.492 |        | 2.382 | 1.763 | 2.189 | 1.1211 |
| 629   | 867   | 191   | 89     | 116   | 023   | 534   | 5.107  | 953   | 74    | 582   | 47     |
| 1.191 | 1.924 | 2.425 | 0.8029 | 1.286 | 2.961 | 1.953 | 1.4243 | 2.783 | 0.681 | 3.528 | 0.4933 |
| 937   | 113   | 166   | 8      | 818   | 882   | 234   | 62     | 719   | 86    | 656   | 18     |
| 3.692 | 1.586 | 4.803 | 2.1201 | 4.314 | 1.466 | 3.143 | 1.3736 | 3.702 | 4.652 | 2.070 | 2.4023 |
| 983   | 588   | 78    | 94     | 494   | 767   | 783   | 74     | 232   | 499   | 534   | 61     |
| 1.684 | 2.117 | 1.026 |        | 0.364 | 1.915 | 2.141 | 2.1618 | 2.398 | 1.564 | 1.721 |        |
| 446   | 991   | 157   | 5.107  | 438   | 257   | 779   | 03     | 368   | 775   | 805   | 5.107  |
| 1.803 | 3.191 | 2.654 |        | 2.961 | 0.704 | 2.696 | 1.2322 | 2.209 | 1.556 | 1.601 | 2.2138 |
| 541   | 923   | 436   | 5.107  | 845   | 21    | 256   | 47     | 012   | 516   | 136   | 1      |
| 2.217 | 1.364 | 0.255 | 1.9772 | 2.450 | 1.823 | 2.259 |        | 2.865 | 1.105 | 1.233 |        |
| 185   | 228   | 681   | 12     | 837   | 29    | 583   | 5.107  | 441   | 934   | 389   | 5.107  |
| 3.158 | 1.882 | 1.656 | 1.2898 | 1.819 | 1.257 | 1.876 | 1.4766 | 1.357 | 4.034 | 1.493 | 2.2595 |
| 38    | 056   | 595   | 57     | 483   | 233   | 204   | 91     | 295   | 043   | 023   | 83     |
| 2.520 | 3.238 | 2.363 | 2.2606 | 3.532 | 1.695 | 3.860 | 1.5009 | 1.556 | 1.219 | 0.213 | 1.8762 |
| 815   | 732   | 489   | 69     | 945   | 871   | 049   | 75     | 248   | 095   | 14    | 04     |
| 2.967 | 1.897 | 0.802 | 1.9132 | 2.136 | 4.096 | 1.424 | 1.4404 | 2.128 | 1.724 | 1.492 |        |
| 042   | 054   | 98    | 2      | 282   | 41    | 362   | 88     | 116   | 023   | 534   | 5.107  |
| 2.298 | 0.468 | 2.120 | 2.1895 | 1.591 | 3.597 | 1.373 | 1.2710 | 1.286 | 2.961 | 1.953 | 1.4243 |
| 446   | 165   | 194   | 82     | 475   | 052   | 674   | 59     | 818   | 882   | 234   | 62     |
| 1.459 | 0.880 | 2.936 |        | 1.341 | 2.994 | 2.161 | 0.8353 | 4.314 | 1.466 | 3.143 | 1.3736 |
| 319   | 196   | 515   | 5.107  | 987   | 698   | 803   | 42     | 494   | 767   | 783   | 74     |
| 1.078 | 2.190 | 3.635 | 2.0705 | 3.460 | 2.332 | 1.232 |        | 0.364 | 1.915 | 2.141 | 2.1618 |
| 483   | 758   | 592   | 34     | 82    | 72    | 247   | 5.107  | 438   | 257   | 779   | 03     |

Table 3.2 shows the simulated samples from Inverse Rayleigh Distribution and HDEWMA control charting statistics calculations.

#### 4. CONCLUSIONS AND RECOMMENDATIONS

In this paper, we have proposed a Hybrid Double Exponentially Weighted Moving Average HDEWMA control chart. We used Monte Carlo Simulations to determine the constants of the control chart for various values. The control chart is proposed for monitoring the inverse Rayleigh lifetimes. The calculated results are for the shifts in scale parameter i.e. 10% - 40% increasing shifts. The result shows that the proposed HDEWMA chart out performs the competitor control charts (i.e. EWMA & CUSUM Charts). The proposed chart performance is assessed for SRS and RSS schemes. It is observed that the HDEWMA using RSS out performs the HDEWMA using SRS, EWMA using RSS, EWMA using SRS, CUSUM using RSS and CUSUM using SRS.

The most efficient results for the proposed chart is obtained for 40% increasing shift. The study can be extended by using median rank set sampling, double rank set sampling and extreme rank set sampling. Since the proposed chart uses the sensitizing rule; single point falls out-of-control so the work can be extended using 2/3, 3/4 out of control sensitizing rule. The similar type of control charts can be developed for other skewed distributions e.g. Rayleigh, Gamma, Inverse Gamma etc.

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